Visualising the Results of a Novel Electoral System

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Abstract

Election results in the United Kingdom are disproportionate to the level of national support held by political parties, which is a feature of the first past the post electoral system used for general elections. With the goal of producing election results that reflect all votes cast by the electorate, a novel electoral system, the Concentrated Vote, was created by Dr. David Sterratt. The aim of this project is to establish whether the results produced by Concentrated Vote improve proportionality compared to first past the post and to develop an accessible interface of visualisations to show the differences between the two systems. This comparison is carried out by applying the Concentrated Vote electoral system to general election results for the United Kingdom from 1979 to 2017. A web based visualisation system, to be understood by a wide audience, comparing the two outcomes was developed. Under the disproportionality indices for electoral systems and other proportionality measures employed, Concentrated Vote resulted in an improvement in proportionality compared to first past the post. The web interface of visualisations, developed using the R Shiny framework, proved to be effective in terms of being easily understood by a general audience in user evaluations. These findings are significant and contribute to the discussion on electoral reform, as it evidences the disproportionality of first past the post, provides a possible solution, and establishes an accessible interface on this topic for the general public.

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Declaration

I declare that this thesis was composed by myself, that the work contained herein is my own except where explicitly stated otherwise in the text, and that this work has not been submitted for any other degree or professional qualification except as specified.

(*Aisling Mac Ardle*)

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Chapter 1

Introduction

The popular vote and election outcomes do not always go hand in hand, a point that has come increasingly into focus in UK and US elections in recent years. A clear example of this phenomenon was Hilary Clinton's failure to secure the US presidency in 2016 despite achieving a majority of the national popular vote. An electoral system is a set of rules governing how votes are cast in an election and how the results of the vote are determined (Smelser et al., 2001). The type of electoral system in use has a significant influence on the outcome of an election.

Certain electoral systems, namely non-proportional systems, do not reflect the volume and variety of votes cast for each party as these systems take only votes for winning candidates into account. First past the post (FPTP) is a non-proportional system used in the UK for general and local elections. This system can create a situation where a party has received a sufficient number of seats in parliament to form a majority government despite not having a majority in terms of the overall national popular vote. In the 2015 general election in the UK the Conservative party formed such a majority government, achieving less than 40% of the popular vote. Arguably, such situations have led to a disconnection between the politics of those elected and the electorate, as voters' views are not fully taken into account.

With the aim of improving the disproportionate relationship between the votes cast for each party and the number of seats won, an alternative electoral system, the "Concentrated Vote", was developed. This project focuses on discovering whether the results produced by this system improve proportionality by applying the Concentrated Vote system to the results of UK general elections from 1979 to 2017 and comparing the outcome against the results of FPTP. This comparison is carried out by analysing and preparing visualisations of the results and developing a web interface to explain

and make them accessible to the public.

This chapter details the motivation behind the project, describes the problem, the hypothesis and project goals, and outlines the remaining chapters in this report.

1.1 Motivation

Non-proportional electoral systems are the oldest and simplest systems in use in democratic states and, despite a number of notable drawbacks, the most popular. Plurality voting, better known as FPTP, is the most commonly used non-proportional electoral system and is often described as "winner takes all". It is used to elect members of parliament (MPs) in the UK general elections to the House of Commons. FPTP produces results that reflect the majority of the population, even where the majority is a slim one. It is often criticised for producing results that disproportionately favour certain political parties and that are unrepresentative of the whole country's voting population. FPTP requires smaller parties, such as the Green Party, to get hundreds of thousands of votes and sometimes millions of votes for just a single seat, whereas as larger parties like the Conservatives require a number in the tens of thousands.

This recurring issue at each general election led to the development of the Concentrated Vote electoral system. The Concentrated Vote is an unpublished electoral system created by Dr. David Sterratt at the University of Edinburgh. The Concentrated Vote is an alternative system to FPTP which aims to produce proportional results while maintaining the major benefit of FPTP, which is its simplicity for voters. Votes are cast as in FPTP, with a single vote and a single MP elected per constituency. Votes for candidates are then shifted between constituencies so they become concentrated based on where support for a particular party is strongest through the use of Lagrange multipliers. The system's aim is to maintain proportionality and reduce the large number of votes wasted by FPTP.

1.2 Problem Description

Prior to this project, the Concentrated Vote had only been applied to the 2010 general election data and minimal analysis had been completed on the results. The aim of this project was to apply the system to general election data from 1979 to 2017 and prepare a web interface of visualisations. The visualisations should convey whether or not the

Concentrated Vote has improved proportionality and provide a comparison with the FPTP results.

1.3 Hypothesis

The hypothesis of this project is that the Concentrated Vote electoral system will produce results that are both proportional and appear reasonable to voters when applied to the UK general election data for the years 1979 to 2017. This approach seeks to avoid the major drawback of the FPTP system, namely the disproportionate relationship between the number of votes cast per party and the number of seats secured per party, while maintaining the major benefit of the system, its simplicity for voters.

1.4 Project Goal and Results

The goal of the project was to apply the Concentrated Vote system to the general election data from 1979 to 2017 and prepare a web interface of visualisations to test the hypothesis of improved proportionality compared to FPTP. Two parts of the project were evaluated: the web interface (available at `maca.shinyapps.io/mscproject/`) and the proportionality of the Concentrated Vote election results. The web interface was evaluated using the think-aloud method and a questionnaire. The Concentrated Vote results were evaluated using political science measures of disproportionality, comparisons with alternative systems, and other proportional measures. The Concentrated Vote proved to produce proportional results by each of the measures used to evaluate it.

1.5 Report Outline

The remaining chapters of this report are structured as follows:

- Chapter 2 explores the literature relating to electoral systems, gives an outline of attempts at electoral reform in the UK, and provides a detailed explanation of the Concentrated Vote electoral system.
- Chapter 3 describes the methodology used to test the hypothesis, including data preparation, application of the Concentrated Vote electoral system, interface design and development, and discussion of the presented visualisations.
- Chapter 4 evaluates and discusses the findings of the application of Concentrated Vote to UK general election data, compares these results against outcomes under

FPTP, and evaluates the web interface.

- Chapter 5 concludes with a discussion of the project findings and possible further work.

Chapter 2

Background

Elections are an essential feature of democracy. The two most popular types of electoral systems in use worldwide can be classified as either majority/plurality or proportional systems. This chapter explores the literature relating to these types of electoral systems and gives an overview of electoral reform in the UK. A detailed technical description of the Concentrated Vote electoral system is also provided.

2.1 Non-proportional Electoral Systems

Non-proportional electoral systems are the earliest and most straightforward systems in use in democratic states. Despite some significant drawbacks they remain the most popular type of electoral system with 35.4% of all countries worldwide using them (Inter-Parliamentary Union, 2016). Plurality voting, which is discussed further below, in particular enjoys continued favour. Outcomes produced in non-proportional systems reflect the perspective of a majority, regardless of the margin of that majority, leading to these systems being described as "winner takes all". A significant critique of these systems is the results they produce disproportionately favour certain political parties and, for this reason, give rise to governments that are unrepresentative of a state's voters taken as a whole. Such outcomes discourage voters from supporting small or new parties because the electoral system underrepresents parties that do not gain a considerable proportion of the vote (Lublin, 2014). The literature has found, however, that regional concentration of support can enable smaller parties to prosper in non-proportional systems (Lublin, 2014).

2.1.1 Plurality Voting

Plurality voting, better known as first past the post (FPTP), is strongly associated with the UK and is used by many former British colonies and Commonwealth states (Norris, 1997). The FPTP system is the most common non-proportional electoral system globally and has been used in the UK for general elections since 1885 following the Third Reform Act of 1884-1885 (ACE, 2017). It aims to create an exaggerated share of seats in the national parliament for the most popular party in order to produce a government with an effective parliamentary majority (Norris, 1997). The system works by splitting a country into constituencies of approximately equal population size, in which a single candidate will be elected. Voters indicate on a ballot the candidate of their choice and the candidate with the most votes in a constituency is elected (Smelser et al., 2001). The only requirement for election with FPTP is that a candidate receives more votes than the other candidates in that constituency.

The major benefits of the system are its simplicity and that results are known promptly, as votes can be counted very quickly. The rationale behind FPTP is that, by awarding a large share of the seats in parliament to one party, effective governance is ensured, which is in contrast to the perceived efficacy of governments that seek to represent minority views. Elections in the UK have generally produced large parliamentary majorities (Farrell, 2011). The 2010 general election, however, did not and the absence of a significant majority resulted in a Conservative/Liberal Democrat coalition, which lasted until 2015. It was shortly followed by the 2017 general election, which resulted in a Confidence and Supply Agreement between the Conservatives and the Democratic Unionist Party (DUP) after the Conservatives failed to win a majority (Conservatives, 2017). Coalition governments are rare in FPTP systems. This rarity is evidenced by Duverger's law, stating that non-proportional electoral systems, specifically referring to FPTP, structured within single member constituencies tend to favour a two-party system and that proportional systems tend to favour multi-partyism (Riker, 1982).

There are a number of significant criticisms of the FPTP system. The primary critique is that the results are not representative of the views of the electorate taken as a whole. Typically, the number of parliamentary seats a party holds does not represent its share of national popular support. This disparity is largely because candidates do not need to win an overall majority of the vote, they need just one more vote than each of the other candidates in the constituency (Farrell, 2011). This thin margin of victory gives rise to a high percentage of wasted votes. Additionally, FPTP does not concentrate

votes on a national level so a party will not succeed in FPTP elections unless it has regional concentrations of support (Farrell, 2011).

2.1.2 Majoritarian Electoral Systems

A majoritarian electoral system requires a winning candidate to have more than 50% of the vote. The requirement to win a minimum threshold of the vote differentiates a majoritarian system from FPTP, which simply requires the winning candidate to have more votes than other candidates (Smelser et al., 2001). The two most common variants of a majoritarian system are the alternative vote and the two-round system, both of which are discussed below.

2.1.2.1 Alternative Vote

Alternative Vote (AV) is a voting system used in single-seat elections. Instead of choosing a single candidate, as with FPTP, voters rank candidates in order of preference. In order to win, a candidate must secure an absolute majority of the available vote. Where no candidate secures over 50% of the vote on first preferences, the candidate with the smallest number of votes is eliminated and their votes are redistributed among the remaining candidates (Norris, 1997). This process is repeated until one candidate secures an absolute majority. AV has the effect of translating a close lead into a decisive majority for the leading party (Norris, 1997). AV is significant in a UK context as in 2011 a referendum on whether to replace FPTP with AV as the electoral system for general elections was rejected by a majority of 68% (McGuinness, 2015). This referendum is discussed in more detail in section 2.4.

2.1.2.2 Two-Round System

The two-round system, also known as second ballot, is a voting system used in single-seat elections, which is commonly used in countries with directly elected presidents such as Austria, France and, Finland (Farrell, 2011). Voters cast a single vote for their chosen candidate. In constituencies where a candidate does not secure more than 50% of the vote, a second election is held which includes only the top two leading candidates from the first election (Smelser et al., 2001). The system's aim is for elected candidates to have the support of the district (Farrell, 2011).

2.2 Proportional Electoral Systems

Proportional electoral systems focus on coordinating the share of the vote parties obtain and the distribution of parliamentary seats (Smelser et al., 2001). The inclusion of minority voices and producing election results that are reflective of the voting population as a whole are critically important, in contrast to the effective governance rationale that underpins FPTP. The makeup, rather than the operation, of a national parliament is its key consideration. Proportional electoral systems are widely used in Europe, with them in use in 58.5% of countries, compared with just 12.3% using non-proportional systems (Inter-Parliamentary Union, 2016).

2.2.1 Single Transferable Vote

Single Transferable Vote (STV) is a proportional voting system that uses ranked votes in multi-seat constituencies (Norris, 1997). It is used in Ireland and Malta for parliamentary elections, as well as for the Australian senate and local elections in Scotland and Northern Ireland. STV works by allowing voters to numerically rank the candidates in their constituency in order of preference. For a candidate to be elected, they must have at least as many votes as required by a quota set for that constituency. The Droop quota, $[\#votes \div (\#seats + 1)] + 1$, is the most commonly used quota that ensures the correct number of candidates are elected in a constituency (Farrell, 2011). If no candidate in the constituency meets the quota based on first preferences, the candidate with the smallest number of votes is eliminated and his or her votes are redistributed based on the second preference votes. This process of eliminating the least popular candidate and redistributing votes is repeated until all seats in the constituency have been filled.

2.2.2 Party List Systems

Party list systems take various forms. The underlying idea is that voters are presented with a list of candidates and seats are assigned to candidates based on their party's share of the vote (Norris, 1997). A party list can be open, where voters can select a preference for a particular candidate, as used in Finland, the Netherlands, Norway, and Italy. Alternatively, a party list can be closed, where voters can only select a preference for a political party and the party will determine the candidate, as used in Germany, Israel, Portugal, and Spain (Norris, 1997).

The rank order on the party list determines the elected candidates but there are many

variations of how this is done. The most common methods are the highest average and the largest remainder. The highest average method involves each party's votes being divided by a series of divisors to produce an average vote. After each divisor is applied, the party with the highest average wins a seat. Its vote is then divided by the next divisor. This process is repeated until all available seats have been filled. The most commonly used formulas are the d'Hondt method, used in Austria, Finland, Spain, and for elections to the European Parliament in the UK, followed by the modified Saintë-Lague method, used in Norway, Sweden, and New Zealand. The d'Hondt method uses divisors 1,2,3,4 etc., while the modified Saintë-Lague method uses divisors 1.4,3,5,7 etc. (Farrell, 2011).

The largest remainder method uses a minimum quota, which can be calculated a variety of ways. In the first round, votes are counted and parties with more votes than the quota are awarded seats. The quota is then subtracted from their overall number of votes. In the second round, the remaining seats are given to the party with the greatest remaining number of votes (Farrell, 2011). The Hare quota, used in Denmark and Croatia, is the simplest quota (Norris, 1997). It divides the total number of valid votes by the total number of seats to be allocated, $\#votes \div \#seats$. The Droop quota, used in South Africa, is set out above in section 2.2.1.

The results produced are more proportional and smaller parties have an easier time securing seats in party list systems than in systems like FPTP.

2.3 The “Concentrated Vote” Electoral System

2.3.1 Overview

The “Concentrated Vote” is an unpublished electoral system created by Dr. David Sterratt at the University of Edinburgh. The Concentrated Vote system is an alternative to FPTP and has been applied to the UK general election data from 1979 to 2017 as part of this project. It aims to address the primary criticism of the FPTP system, which is the disproportionality between election results and national political party support, while maintaining the major benefit of FPTP, simplicity for voters. As in the FPTP system, voters indicate the candidate of their choice on a ballot, with a single candidate to be elected in each constituency. Votes are counted and recorded in an array of votes per party in each constituency. However, candidates are only elected after all votes in the country or region are counted and support has been concentrated in the following way. The candidate with the smallest number of votes in a given constituency is eliminated

first. Their votes are shifted to a constituency where the eliminated candidate's party has more votes. The system repeats this process of eliminating candidates one-by-one, resulting in a concentration of votes where a party started out strong. When a single candidate remains in a constituency, they are deemed to be elected at that point. At each stage of this process, the number of votes per constituency and votes per party remains fixed. The process terminates when there is no candidate that can be eliminated without breaking a constraint relating to the number of votes per constituency and votes per party.

2.3.2 Technical Details

The problem can be formulated as a non-convex optimisation problem. The number of votes for party j in constituency i is represented as V_{ij}. These votes are transferred between candidates to give $X_{ij,k}$ votes for party j in constituency i at time stage k. The system aims to transfer votes between constituencies to make them as distinct from each other as possible, while keeping the number of votes per party and per constituency fixed. This is formulated as follows:

$$\text{maximise} \quad \frac{1}{2}\sum_i \sum_j X_{ij,k}^2 \tag{2.1}$$

$$\text{subject to} \quad \sum_j X_{ij,k} = \sum_j V_{ij} \tag{2.2}$$

$$\sum_i X_{ij,k} = \sum_i V_{ij} \tag{2.3}$$

$$X_{ij,k} \geq 0 \tag{2.4}$$

The constraint in equation 2.2 maintains the number of votes per constituency, equation 2.3 maintains the number of votes per party, and equation 2.4 ensures the number of votes for each candidate in each seat remain positive.

Solving this optimisation problem is NP-hard (Manyem and Ugon, 2012), so a plausible, possibly sub-optimal, solution can instead be implemented. *Lagrange multipliers* are used to solve this problem. The method incorporates the maximisation function and the constraints into a *Lagrange function, L*, in such a way that the extreme value is obtained only when the constraints are satisfied (Garrett, 2015). The Lagrange Multiplier theorem allows the translation of the original constrained optimisation problem into a system of simultaneous equations, at the expense of adding an additional variable per

constraint (Gordon, 2015).

This can be formulated by introducing the binary variable R_{ij}, which equals 1 if party j is still in the running for a seat in constituency i, or 0 if they have been eliminated. The total constituency votes are represented by $c_i = \sum_j R_{ij}X_{ij,k} = \sum_j V_{ij}$ and similarly, the total party votes are represented by $p_j = \sum_i R_{ij}X_{ij,k} = \sum_i V_{ij}$.

R is added to the objective function in equation 2.1 and the constraints in equations 2.2 and 2.3 will be replaced with the following constraints:

$$g_l(\mathbf{X}) = \sum_j RX_{lj} - c_l = 0 \tag{2.5}$$

$$h_m(\mathbf{X}) = \sum_i RX_{im} - p_m = 0 \tag{2.6}$$

This system can be defined compactly as the Lagrangian, L, with multipliers λ_l and μ_m for constituencies and parties respectively:

$$L(\mathbf{X}) = \frac{1}{2}\sum_i\sum_j RX_{ij}^2 - \sum_l \lambda_l g_l(\mathbf{X}) - \sum_m \mu_m h_m(\mathbf{X}) \tag{2.7}$$

with the additional constraint:

$$\sum_m \mu_m = 0 \tag{2.8}$$

This constraint is required to ensure that there is a unique solution for the multipliers.

The gradient of L with respect to X_{ij} is:

$$\frac{\partial L}{\partial X_{ij}} = R(X_{ij} - \lambda_i - \mu_j) \tag{2.9}$$

From this, we want the direction of the gradient of the Lagrangian, L, to be perpendicular to the normal of the constraint. Solving equation 2.10, equation 2.11 can be derived to achieve this for the constituency constraints.

$$\sum_i\sum_j \frac{\partial L}{\partial X_{ij}} \cdot \frac{\partial g_l}{\partial X_{ij}} = 0 \tag{2.10}$$

$$c_l - \lambda_l \sum_m R_{lm} - \sum_m \mu_m R_{lm} = 0 \tag{2.11}$$

Following the same analogy, equation 2.12 can be derived for the party constraints. See

appendix A for full derivations.

$$p_m - \sum_l \lambda_l R_{lm} - \mu_m \sum_l R_{lm} = 0 \tag{2.12}$$

To solve this system of equations for λ_l and μ_m, matrix inversion is employed as follows. Equations 2.8, 2.11, and 2.12 are combined in a matrix equation:

$$\mathbf{A} = \begin{pmatrix} \mathbf{L} & \mathbf{R}-\mathbf{1} \\ \mathbf{R^T} & \mathbf{M} \end{pmatrix} \tag{2.13}$$

$$\mathbf{A} \begin{pmatrix} \vec{\lambda} \\ \vec{\mu} \end{pmatrix} = \begin{pmatrix} \vec{c} \\ \vec{p} \end{pmatrix} \tag{2.14}$$

where $\mathbf{L}$ is a diagonal matrix whose non-zero elements are $L_{ii} = \sum_j R$, $\mathbf{M}$ is a diagonal matrix whose non-zero elements are $M_{jj} = \sum_i R$, $\mathbf{R}$ has elements R_{ij}, $\mathbf{1}$ is a matrix of the same size as $\mathbf{R}$ with every element equal to 1, and $\vec{\lambda}$ and $\vec{\mu}$ refer to λ_l and μ_m respectively. The candidacy ij, for which $X_{ij} = 0$, is found when the direction of the gradient, obtained above, is followed, by computing $s_{ij} = \frac{-X_{ij}}{\frac{\partial L}{\partial X_{ij}}}$ and setting s to be the minimum positive value of s_{ij}. For this ij, $X_{ij} = 0$ and R are set and these calculations are repeated. If the gradient equals zero, within a tolerance, the process will end. The tolerance is the absolute value of the maximum column of the matrix, which contains the difference between the column totals of the original vote matrix and the shifted vote matrix, which must be less than 1, see equation 2.15. This ensures the positivity of votes without explicitly including the constraint.

$$\max_j |p_j - \sum_i X_{ij}| < 1 \tag{2.15}$$

2.3.3 Worked Example

Algorithm 1 presents a pseudocode description of how the Concentrated Vote electoral system works, demonstrating how the equations discussed above are applied in a programming environment. An example will be used to demonstrate how the system works in practice with reference to the pseudocode of the *concentrator* function, the function that applies the Concentrated Vote electoral system. Consider the following example in table 2.1, with three parties, the Progressives, the Regressives and the Obsessives, and three constituencies, Smoketown, Verdant Valley and Lily Grove. Under FPTP the Regressives would win all three seats, two of which would be marginal

wins.

Algorithm 1 The Concentrated Vote Electoral System

function CONCENTRATOR(votes) ▷ Where votes is a matrix of the FPTP results
$V \leftarrow votes$
$X \leftarrow$ initial shifted vote matrix V
$M \leftarrow$ number of constituencies
$N \leftarrow$ number of parties
$R \leftarrow$ remaining vote matrix ▷ MxN matrix with all elements set to 1
while true **do**
$A \leftarrow$ constraint forces ▷ Equation 2.13
$lambda \leftarrow$ constituency multiplier ▷ Obtained by solving equation 2.14
$mu \leftarrow$ party multiplier ▷ Obtained by solving equation 2.14
$G \leftarrow$ gradient ▷ Equation 2.9
if all elements of $G = 0$ **then**
break
end if
$sij \leftarrow$ -X / G ▷ distance from X to intersection of line in direction of gradient to the first constraint to be encountered
$s \leftarrow$ minimum positive value of sij
$elimij \leftarrow$ position of sij in matrix
$X1 \leftarrow$ X + G * s ▷ Update matrix X to eliminate/elect candidate(s)
if discrepancy in party totals **then** ▷ Equation 2.15
break
end if
$X \leftarrow X1$
$R \leftarrow$ set position of eliminated candidate $elimij = 0$
end while
return X ▷ Matrix of shifted votes
end function

FPTP Votes	Progressives	Regressives	Obsessives
Smoketown	48	90	12
Verdant Valley	61	70	12
Lily Grove	60	64	10

Table 2.1: Example Votes Cast - Corresponds to *V*

Before the loop begins, the variables *V, R, X, M, N* are initialised based on the votes matrix passed to the function, which in this case is a matrix of the information in table 2.1. The loop will then begin and run until a constraint becomes broken causing a break in the loop. The constraint forces are then calculated as in equation 2.14 by *A*, *lambda*, and *mu*, as is the gradient in equation 2.9, which is denoted *G*. If the gradient has converged and equals 0, the loop will be terminated. In this case, however, the

gradient is not 0 and the process of eliminating a candidate will begin. The distance in the direction of the gradient from the position of the current votes, *sij*, is calculated by $\frac{-X}{G}$, and the minimum positive value, *s*, is chosen. The *s* value is then used, with the gradient, to update the matrix *X* which will cause single or multiple candidates to be eliminated and/or elected.

Following this first iteration of the loop, the Progressive candidate in the Smoketown constituency is eliminated. As a result, their votes will be shifted to the Progressive candidate in the other constituencies and votes for the other parties will be shifted to Smoketown in order to maintain the total number of votes per party and per constituency. Table 2.2 displays the resulting matrix, *X1*, from this process. If, following the previous step, there is a discrepancy in a party's total number of votes, the loop will be terminated. The position of the Progressives candidate in Smoketown in the **R** matrix will be updated from 1 to 0 to indicate the candidate's elimination.

Iteration 1	Progressives	Regressives	Obsessives
Smoketown	0	146	4
Verdant Valley	81	48	14
Lily Grove	88	30	16

Table 2.2: Example Votes after 1 Iteration of the Loop

Since none of the constraints have been violated, another iteration of the loop will occur. The process will repeat as described above, and table 2.3 displays the resulting matrix, *X1*. This update caused the Obsessive candidate in the Smoketown constituency to be eliminated, leaving just a single candidate left standing in Smoketown and, therefore, the Regressive candidate is elected in this constituency. Again, as no constraints were broken a third loop will be completed, where the Regressive candidate is eliminated in Lily Grove, resulting in the matrix displayed in table 2.3. As before, the position of this candidate in the **R** matrix will be updated from 1 to 0.

Iteration 2	Progressives	Regressives	Obsessives
Smoketown	0	150	0
Verdant Valley	80	47	16
Lily Grove	89	27	18
Iteration 3	Progressives	Regressives	Obsessives
Smoketown	0	150	0
Verdant Valley	62	74	7
Lily Grove	107	0	27

Table 2.3: Example Votes after 2 and 3 Iterations of the Loop

As the constraints remain satisfied, a fourth iteration of the loop will be performed, causing the Obsessive candidate in Lily Grove to be eliminated. As only a single candidate is left in Lily Grove, the Progressive candidate is elected. As there is now only a single constituency remaining with a candidate to elect and no place to shift votes, the loop will terminate and the matrix displayed in table 2.4 will be returned. The winning Concentrated Vote candidates are then chosen as the candidate with the most votes in a constituency, which is the same method as FPTP albeit with a different matrix of votes.

Iteration 4	Progressives	Regressives	Obsessives
Smoketown	0	150	0
Verdant Valley	35	74	34
Lily Grove	134	0	0

Table 2.4: Example Votes after the Final Iteration

Although this example was simple, the candidates elected by FPTP and Concentrated Vote differ. FPTP elected the Regressive candidates in all three constituencies, despite two of the wins being marginal. The Concentrated Vote elected two Regressive candidates and a Progressive candidate in the Lily Grove constituency. The way in which the Concentrated Vote concentrates support for a party across all constituencies in the constituency where that party started out stronger, is demonstrated by the differences between the initial votes in table 2.1 and the final Concentrated Votes in 2.4.

2.4 Electoral Reform in the UK

2.4.1 Early Attempts

While the FPTP system has long been in use in the UK, there have been several attempts at electoral reform, attempting to curb some aspects of its operation. The first main attempt to make changes to voting rules for elections to the House of Commons was from the mid-nineteenth century to the early 1930s. In a series of debates, the three systems that featured were the limited vote, AV, and STV. STV gained the widest recognition and was sought, unsuccessfully, to be introduced in the 1860s (Farrell, 2011). Following the introduction of FPTP, the Proportional Representation Society (the predecessor of the Electoral Reform Society) was formed in the 1880s with the aim of lobbying for STV in UK elections.

The next significant reform attempt began in the early 1970s, largely in reaction to the disproportionate election results in 1974 where Labour secured more seats

than the Conservatives despite having fewer votes. In 1975 the Hansard Society set up a Commission on Electoral Reform, which produced a report proposing a mixed-member electoral system, and an all-party National Committee for Electoral Reform was established (Farrell, 2011). As a result, electoral reform remained on the national agenda until the 1980s. However, as electoral reform was primarily the concern of smaller parties and minorities, neither Labour nor the Conservatives were prepared to drive it forward, in fear of losing a single-party majority (Farrell, 2011).

Following the election of a new Labour government in 1997, breaking 18 years of uninterrupted Conservative rule, the idea of electoral reform began to gain national attention again after being placed high on Labour's agenda (Farrell, 2011). While an independent Commission on Electoral Reform was established, its report on electoral reform was unconvincing and the Labour manifesto promise of a referendum on electoral reform soon evaporated (Farrell, 2011).

2.4.2 Alternative Vote Referendum 2011

At the 2010 general election, Labour proposed holding a referendum on replacing FPTP with Alternative Vote (AV) within 18 months (Curtice, 2013). After Labour failed to win the election, the new government, a coalition between the Conservatives and Liberal Democrats, put the proposition that FPTP should be replaced with AV to popular vote. The proposal was rejected by the electorate, with 67.9% rejecting the replacement of FPTP (McGuinness, 2015).

The Yes campaign was backed by the Liberal Democrats. As support for the Liberal Democrats tends to be evenly spread geographically, they are continuously under-represented in the national parliament by FPTP and would have gained seats from the introduction of AV. By the time of the 2011 referendum, however, the Liberal Democrats, and in particular their leader, then deputy prime minister, Nick Clegg, were unpopular in the national media and were often blamed for the shortcomings of the coalition government. The Yes campaign was also backed by the Electoral Reform Society. Arguably, backing AV was a compromise for the Electoral Reform Society as it was not seen as the ultimate aim of electoral reform but rather a halfway house on the road to STV, which was a difficult sell. The No campaign was officially backed by the Conservatives and by a number of Labour MPs, as Labour did not adopt an official position. The Conservatives claimed FPTP delivered greater accountability of governments and reduced the number of hung parliaments (Sparrow, 2011).

Commentators have speculated on the reasons why the electorate rejected AV. Firstly, the referendum offered a choice between two non-proportional electoral systems, rather than a choice between substantially different systems, such as majoritarian and proportional. Secondly, the Yes campaign failed to generate enthusiasm for a system that its supporters themselves had doubts over (Curtice, 2013). Notably, Nick Clegg had previously dismissed AV as a "miserable little compromise", comments which were frequently repeated in national media during the referendum campaign (Clark, 2011). Additionally, many in the Yes campaign saw AV as a halfway house on the road towards a proportional electoral system, a position for which it was difficult to generate public enthusiasm and acceptance. Thirdly, the Yes campaign lacked the support of a major political party. In a system with two large parties, arguably the support of at least one is necessary to tap into grass roots level and local support. Finally, AV was viewed in the media as overly complicated and difficult to understand, which played poorly against the simplicity of FPTP. For example, the Electoral Commission's booklets on the referendum explained FPTP in a few sentences, whereas AV was given three diagrams over several pages.

2.5 Summary

This chapter has set this project in context. It explained the major types of electoral systems in use worldwide and discussed the advantages and disadvantages of FPTP, the electoral system used in the UK. An overview of Concentrated Vote, its aims, and a worked example were given. The history of electoral reform in the UK was also outlined.

Chapter 3

Methodology

Chapters 1 and 2 introduced the Concentrated Vote electoral system, explained the motivation underpinning the development of a new electoral system, and explored the relevant literature. This chapter describes the methodology used to test the hypothesis set out in section 1.3. This methodology includes data preparation, application of the electoral system, interface design and development, and discussion of the visualisations created for this project. Subsequent chapters will evaluate and discuss the findings of the Concentrated Vote and the user interface.

3.1 Initial Setup

3.1.1 Data Requirements and Sources

In order to compare the results of the Concentrated Vote electoral system against the results of FPTP, the complete results of UK Parliament general elections for the years 1979, 1983, 1987, 1992, 1997, 2001, 2005, 2010, 2015, and 2017 were required. The complete result for each year is a dataset containing the number of votes received for each candidate, i.e. not simply just the winning candidate, in each constituency in the UK. The results were obtained from the following sources.

The Electoral Commission is an independent body set up by the UK parliament in 2001 to regulate party and election finance and to set independent standards for elections. Complete electoral data for the years 2005, 2010, and 2015 is available to download from its website in CSV format (Electoral Commission, 2017). Electoral data for the years 1971 to 2001, i.e. before the formation of the Commission, and electoral data for the recent June 2017 election was not available from the Commission at the

time of writing. Furthermore, the complete data for these years is not available from the House of Commons library or the Parliamentary Archives; only partial data and summary statistics of the results are available. Further research was required to collect the data for these years from a variety of other resources.

Richard Kimber's Political Science Resources website is a collection of links to the politics and government of the UK and the US. From here, complete electoral data for the years between 1979 and 2001 was obtained in XLS format (Kimber, 2017), where the data was originally acquired from the Daily Telegraph's Election Supplement in the corresponding year.

Britain Elects is a poll aggregation service with the aim of delivering unbiased commentary and analysis of British politics and public opinion. They released partial electoral data for 2017 general election in CSV format (Britain Elects, 2017). The data was partial in that it contained only the electoral results for England, Scotland, and Wales, as well as containing an "other" column that grouped smaller parties together. To complete the dataset, the results for Northern Ireland were added manually. This information was obtained from detailed listings on the Belfast Telegraph's website (Belfast Telegraph, 2017).

3.1.2 Development Environment

The objective of the project is to apply the Concentrated Vote to historic electoral data and to visualise and analyse the results, displaying them on a web interface. The project required a software programme to carry out this analysis and create the visualisations. R was chosen for the development because the Concentrated Vote algorithm was developed using R and it has powerful visualisation capabilities.

R is an open source programming language and environment for statistical computing and graphics (R Project, 2017a). R is available under the GNU General Public License and is used widely for statistics, data analysis, and data visualisation. The functionalities of R are extended greatly by packages, which can be installed to add specialised functions (R Project, 2017a). The R packages *ggplot2*, used to create the visualisations, and *shiny*, used to develop the interface, are discussed in detail in section 3.4.1.

RStudio Desktop is a free and open source graphical integrated development environment for R (R Project, 2017c). RStudio version 1.0.153 with R version 3.4.0 was used to apply the Concentrated Vote system to the electoral data as well as for the

development of the entire web interface.

3.2 Data Preparation

Prior to the design and development of the web interface, the datasets, containing the general election results, were cleaned and pre-processed. The Concentrated Vote and FPTP electoral systems were then applied to the data. The results from this application were used to prepare the visualisations.

3.2.1 Description of Raw Datasets

A total of ten raw datasets were involved, a dataset for each general election between the years of 1979 and 2017, obtained from three different sources discussed in section 3.1.1. General elections in the UK occurred in the following years: 1979, 1983, 1987, 1992, 1997, 2001, 2005, 2010, 2015, and 2017.

As the datasets were from different sources, the level of detail varied between the datasets. The only information required for this project was the votes received by each candidate in each constituency. In the three most recent elections (2010, 2015, and 2017), there have been 650 constituencies. Constituency boundaries, however, have been updated numerous times between 1979 and 2017. The number of constituencies has varied between 635 and 659 in this period.

3.2.2 Data Cleaning and Pre-processing

An Excel VBA (Visual Basic for Applications) Macro was created to clean and transpose all datasets to prepare them for working in R. An exception was the 2010 dataset, which was already in the transposed format. The format in which the Concentrated Vote algorithm in R required the electoral data was a row per constituency, with each column containing the votes received for a particular party. The majority of the raw datasets contained all parties in a single column and a corresponding column with all votes received. The Macro transposed the datasets into a standard format and removed irrelevant columns. Once all data was in a standard format the process of applying the Concentrated Vote electoral system to multiple years was straightforward. To ensure the data remained consistent, the total number of votes was cross-checked with the raw data.

Following this procedure, the CSV files were read into the RStudio environment for pre-processing. The first part of the pre-processing involved separating individual independent candidates out. As these candidates were not part of a single party and ran independently of each other, it did not make sense to include them in a single column, as they are not related to each other. They were separated so that each independent candidate has their own column in the data matrix.

There are often political parties that run in the general election and receive a minuscule proportion of the vote. For example, in 2010 139 parties stood in the election, with many of these parties receiving only a few hundred votes, and only 12 parties winning seats. As parties with so few votes will never win a seat, they were removed from the data matrix to improve the speed at which the Concentrated Vote algorithm could run. The cut-off used was 50% of the total votes in the smallest constituency in the respective year. This cut-off was chosen as, theoretically, the candidate that wins a seat should have the majority of the constituency where they win the seat. If a party does not have enough votes for a majority, i.e. more than 50%, in the smallest constituency, they should not be able to win a seat and so the party was removed.

3.2.3 Application of Electoral Systems

The application of the both electoral systems, FPTP and the Concentrated Vote, took place in RStudio following the data cleaning and pre-processing. The FPTP system was applied to the electoral datasets, as opposed to looking up the actual results, for simplicity and speed. To apply the FPTP system to the data, simply the party with the highest votes in each constituency was the winner, that is, the maximum column (party) in each row (constituency) of the data matrix. This can be calculated rapidly and easily, saving time searching for pre-calculated results. To ensure data integrity, the total number of seats won per party was spot checked to ensure no errors were incurred.

To apply the Concentrated Vote system, the algorithm developed in an R script by Dr. David Sterratt was applied. The R script executes the process described in section 2.3.2, using a function called *concentrator*. Following the cleaning and pre-processing of the datasets, the datasets for each year were separated into two data frames: one for Northern Ireland and one for the rest of the UK. This was done as the parties running in Northern Ireland are usually distinct from the rest of the UK; in some years there was no overlap of parties between the two. This results in essentially two “islands” of constituencies in the matrix $\mathbf{R}$ in the concentrator function, and with no link, the matrix

in equation 2.14 has an infinite determinant, hindering the execution of the function. The concentrator function was then applied to the data frames for each election year individually. The result of this function was a matrix of shifted votes per party for each constituency. To calculate the seat winners, as in FPTP, the party with the most votes in each constituency won the seat. Again, this was completed by taking the maximum column in each row of the data matrix. The results the FPTP and Concentrated Vote systems were stored in data frames, which were used in the preparation of the web interface.

3.3 Web Interface Design

3.3.1 Design Rationale

Usability and comprehensibility of the visualisations were the underpinning ideas for the design of the interface. Usability involves ensuring the interface is easy and effective to use, and enjoyable from the user's perspective (Rogers et al., 2011). The goal of the design was to create a positive user experience that was enjoyable, motivating, and engaging. A human-centred design was employed, which Norman (1988, p.8) defines in his book The Design of Everyday Things as "an approach that puts human needs, capabilities, and behaviour first, then designs how to accommodate those needs, capabilities, and ways of behaving". To achieve this, a number of key design principles, including visibility, consistency, and affordance, discussed by Norman (1988), were followed.

3.3.1.1 Visibility

Visibility was important in the design of the interface, as it is essential the user is informed and understands how to use the interface. Each webpage of visualisations has a heading clearly indicating the topic depicted in the visual, as well as a more detailed text description. This display keeps the user aware of the section of the interface they are in, avoiding disorientation. The main menu banner, which is spread across the top of the interface on each page, further increasing visibility, allows the user to return to a previous webpage at any point. It allows a user to find their way back to the home page and avoids the user getting lost.

3.3.1.2 Consistency

Creating a consistent interface design means it is easier to learn and use (Rogers et al., 2011). A consistent colour scheme, layout, and flow were employed for this purpose. Aesthetically pleasing colours, that were not overpowering, were chosen for the visualisations, specifically familiar colours that represent the political parties to ease recognition. The simplicity of the white background throughout the interface emphasises clarity and minimalism. Black text against a white background was chosen as this contrast makes the text easy to see and read. The aim for the layout of the interface was to keep it as consistent as possible and to avoid overcrowding it with information. Each the webpages were designed to have a minimalist layout, with text evenly spaced and easy to read and understand. The main menu and the year selection dropdown menu are located in the same position on each webpage.

3.3.1.3 Affordance

Norman (1988) explains the design principle of affordance as being simply to guide the user. When the affordance of an interface is perceptually obvious it is easy to know how to interact with it (Rogers et al., 2011). This was taken into account when designing the interface, as it is structured and works like the many popular websites, with no complex or difficult to understand tasks.

3.3.2 Interface Evaluation

As usability and comprehensibility are of central importance to the interface, two methods were used for evaluation. A questionnaire, a series of questions designed to elicit specific information (Rogers et al., 2011), was used for usability testing and the think-aloud technique, where users speak aloud their thoughts as they interact with the interface, was used to test both usability and comprehensibility. Well-designed questionnaires obtain clear answers to specific questions and, particularly if prepared online, can reach a large audience. Questionnaires are often used in conjunction with another technique to confirm conclusions (Rogers et al., 2011) and in this project, in conjunction with the think-aloud technique.

3.3.2.1 Think-Aloud Technique

The think-aloud technique was performed with three participants. The participants were asked to say whatever comes into their mind as they view and comprehend the visualisations in the interface. The details of their thoughts and comments were recorded and incorporated into updating the interface, for example, where users found a visual or description unclear or confusing. The full details of the results from the think-aloud are contained in chapter 4, section 4.1.1.

3.3.2.2 Questionnaire

A questionnaire was prepared using Google forms and distributed to a group of 12 students who had spent time using the interface. The questionnaire focused on evaluating user satisfaction and was based on the Questionnaire for User Interaction Satisfaction (QUIS) tool developed at the University of Maryland which aimed to measure the user's overall system satisfaction (Chin et al., 1988). The results of the questionnaire are discussed in detail in chapter 4, section 4.1.2.

3.4 Web Interface Development

This section describes the development of the web interface, built using R Shiny. R Shiny applications differ to standard web development in that they are build around inputs and outputs, with the inputs in the UI passed through to the server, described in more detail below. This web interface consists of two types of webpages: a homepage and underlying pages. The homepage acts as the "anchor" of the website, while the underlying pages provide more detailed content (Shelly and Campbell, 2012). The interface can be viewed for election years 1979 to 2017 at `https://maca.shinyapps.io/mscproject/`, or, appendix B contains screenshots of each of the webpages for the 2015 general election.

3.4.1 R Packages

The R package *ggplot2* was used to create the majority of the graphics and visualisations of the electoral data. The ggplot2 package was developed by Hadley Wickham, a prominent R developer (Wickham, 2010). It was chosen to prepare the visualisations as it offers a powerful graphics language with the capability of creating complex yet aesthetically pleasing visuals.

The R package *magick* was used to create animated Graphics Interchange Format (GIF) images of the electoral data. The package allows advanced image processing in R and wraps *ImageMagick*, which is a comprehensive open source software suite for image processing (R Project, 2017b).

The R package *Shiny* was used to create the web interface. The Shiny package is a web application (app) framework for R. An R shiny app can be built entirely using R, or HTML, CSS, and JavaScript can be used for additional flexibility (RStudio, 2017a). Shiny uses a reactive programming model, which means that complicated event handling code is not necessary. Shiny apps are made up of two components: a user interface (UI) definition and a server script (RStudio, 2017a). The UI component is made up of HTML and contains information relating to the appearance of the app, such as the layout and menus, while the server component contains the instructions of what the server will execute. R shiny applications are built around inputs and outputs, with the inputs in the UI passed through to the server, which is then to used to produce the end result of what is displayed on the interface.

3.4.2 Hosting and Deployment

A Shiny app can be run locally on a computer easily through the command line but this limits the audience greatly. To reach a greater audience, Shiny apps can be deployed to a web server, either to a self-hosted Shiny server or a Shiny server hosted by *shinyapps.io*, which is a subscription based hosting service run by RStudio.

Shinyapps.io was chosen as the most appropriate option for the following reasons. Shinyapps.io is extremely easy to use. A Shiny app can be deployed from RStudio to the web within seconds, with no hardware or installation required. Additionally, shinyapps.io is secure and scalable (RStudio, 2017b).

As shinyapps.io is a subscription based hosting service, there are a number of account types. A free account is available, however the number of active hours, being the hours the application is being used, is very limited. A starter account is available, for a fee of $9 a month, with a less restrictive number of active hours. A number of additional professional account types with more features and a higher fee are also available. However, the starter account is the most suitable in this case.

3.4.3 Description of Visualisations

Each webpage of the interface looks at a different element of proportionality to test whether the hypothesis holds and each will be discussed in turn. All visualisations described in this section are available on the web interface for all election years between 1979 and 2017, with the exception of the proportional maps. The proportional maps are only available for the years 2010, 2015, and 2017 as the constituency boundaries were different prior to 2010. The coordinates of the proportional maps were obtained from Dr. Benjamin Hennig, a geographer at the University of Iceland (Hennig, 2017). The visualisations in the following section will be discussed with reference to the 2015 general election because the results are often described as "the most disproportionate in history" (Garland and Terry, 2015), and therefore demonstrate the difference in results most effectively. The results of other electoral years follow the same patterns unless otherwise stated.

3.4.3.1 Homepage

The homepage is the initial page a visitor sees when the web interface is opened. On a well-designed website, the homepage should clearly indicate who owns the website, what a visitor can expect, why they should visit the website, and where information and features are located (Shelly and Campbell, 2012). The homepage of the web interface provides an explanation of the Concentrated Vote system with a graphic demonstrating how the process works on a proportional map of the UK, as seen in figure 3.1. The background and motivation behind the Concentrated Vote is explained, as well as a high-level description of the system and this project. A detailed description of the system is not included on the homepage for usability purposes, as the details are highly mathematical. Instead, to avoid alienating some users with this technical information, an overview of the system is provided with an option to view further details of the system if the user wishes.

The graphic displayed on the homepage of the interface is an animated GIF image showing Concentrated Vote in operation. A GIF is a format of images, which can be animated or static, and in this case is animated, demonstrating the process of shifting votes between constituencies until it reaches a stage where no candidates can be eliminated without breaking one of the constraints relating to the number of votes per constituency and per party. The animated GIF is available for the years 2010, 2015, and 2017 only, as the constituency boundaries were different prior to 2010. The user can select the

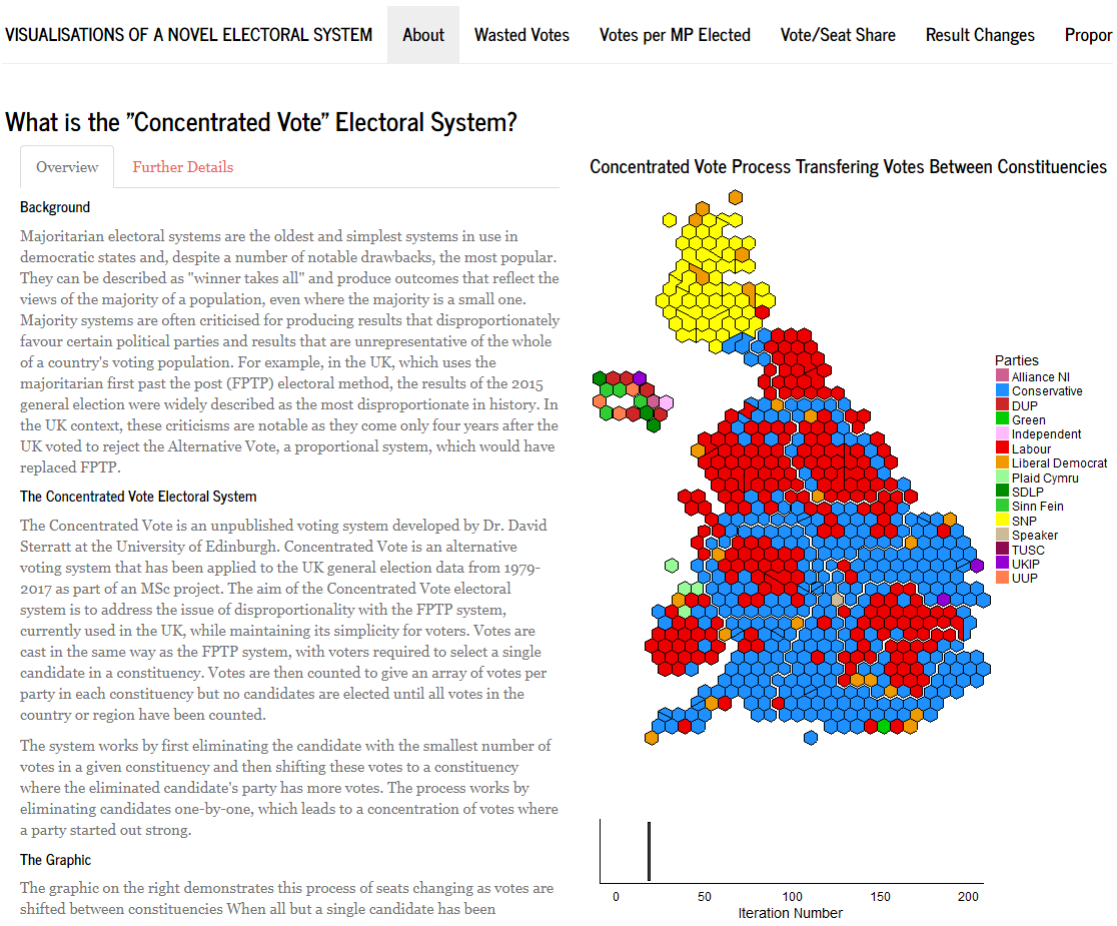


Figure 3.1: Homepage of Web Interface

year using a dropdown menu on the top righthand corner of the interface (not visible in figure 3.1, see appendix B). The animated GIF is a proportional map, formally known as a cartogram, which shows winning seats in constituency shifting throughout the process through a change in colour to that of the winning party. There is also a bar graphic accompanying the cartogram to show the stage of the process the animated GIF is at. A cartogram is a map that displays geographic regions in proportion to their population or another property (Gastner and Newman, 2004). The cartogram used on the homepage represents each constituency as a hexagon, coloured by the party colour of the winning party at a given stage in the Concentrated Vote process.

To create the GIF of the Concentrated Vote system shifting votes between constituencies on a proportional map, it was recorded each time a seat winner changed while the Concentrated Vote system was running. A cartogram was created, using the ggplot2 package, for each point of change. To create the cartogram, a Shapefile of the coordinates of each hexagon was read into R as a spatial object and using the *fortify* function it was converted to a standard data frame with polygon vertices and attribute

data (Dervieux, 2017). In this format, the ggplot2 package can render polygons using the *geom_polygon* function to plot the cartogram. The cartograms were output as images for each point of change in the Concentrated Vote system and the magick R package then bundled these images together in chronological order to create an animated GIF.

3.4.3.2 Wasted Votes

In the FPTP system, wasted votes are votes for a losing candidate or surplus votes for the winning candidate (Garland and Terry, 2015). As votes for losing candidates are not taken into account in any way, a huge number of votes are wasted. In the Concentrated Vote system, wasted votes are those left over when the process of elimination cannot continue without breaking one of the constraints relating to the number of votes per constituency and votes per party. The amount of votes wasted with Concentrated Vote is significantly lower than the number wasted with FPTP.

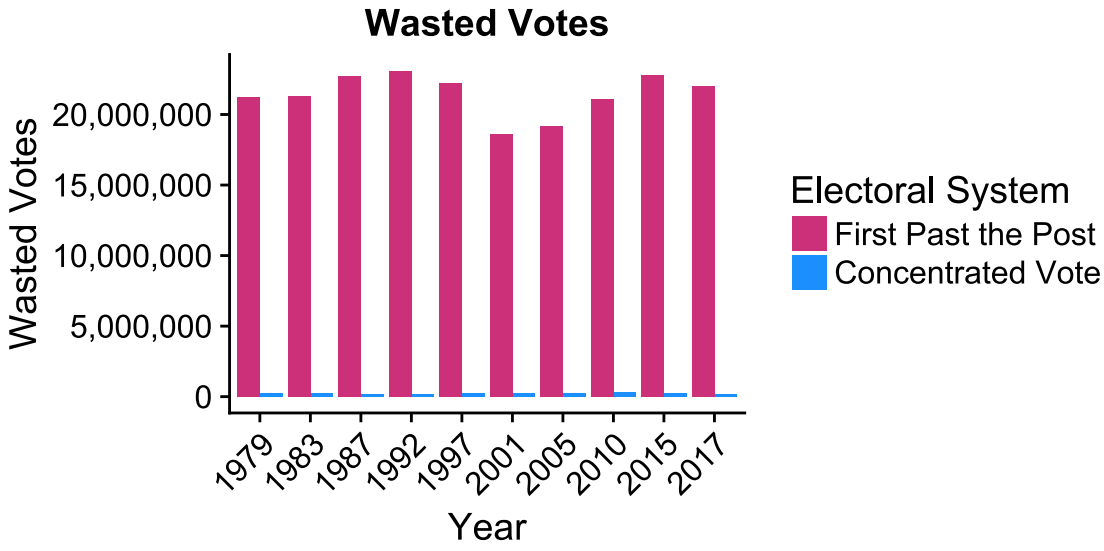


Figure 3.2: Wasted Votes Graphic

The web interface displays this information through a bar chart, using the *geom_bar* function in the ggplot2 package, of the two electoral systems side by side, as well as a detailed table of the wasted votes and percentages for each year. From figure 3.2, a striking difference in the number of wasted votes in the two systems can be seen. In the most recent general election in 2017, the FPTP system wasted approximately 68% of the votes cast, which is 22 million votes. In comparison, the Concentrated Vote system wasted only 0.51% or 166,000 votes. The 2015 general election was a particularly disproportionate year for the FPTP system as parties such as UKIP (UK Independence

Party) had an even spread of support in the UK, which did not translate into seat wins in FPTP. This led to 74% of votes cast, or 23 million votes, being wasted, whereas the Concentrated Vote wasted just 0.76% or 234,334 votes.

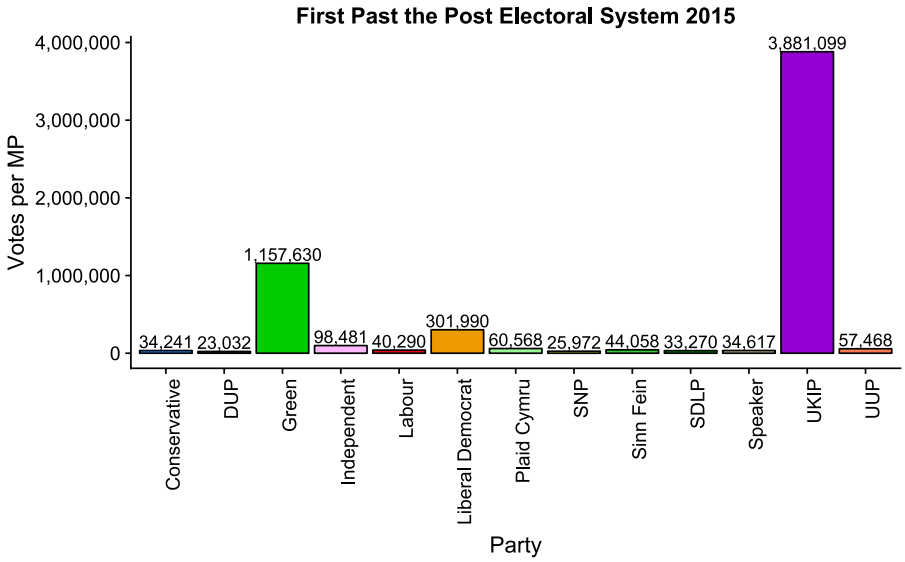
This information provides strong evidence that the Concentrated Vote would improve proportionality. As a significantly lower percentage of votes would have been discarded under Concentrated Vote, the amount of votes that have "no say" in the election outcome has been significantly reduced.

3.4.3.3 Votes per MP Elected

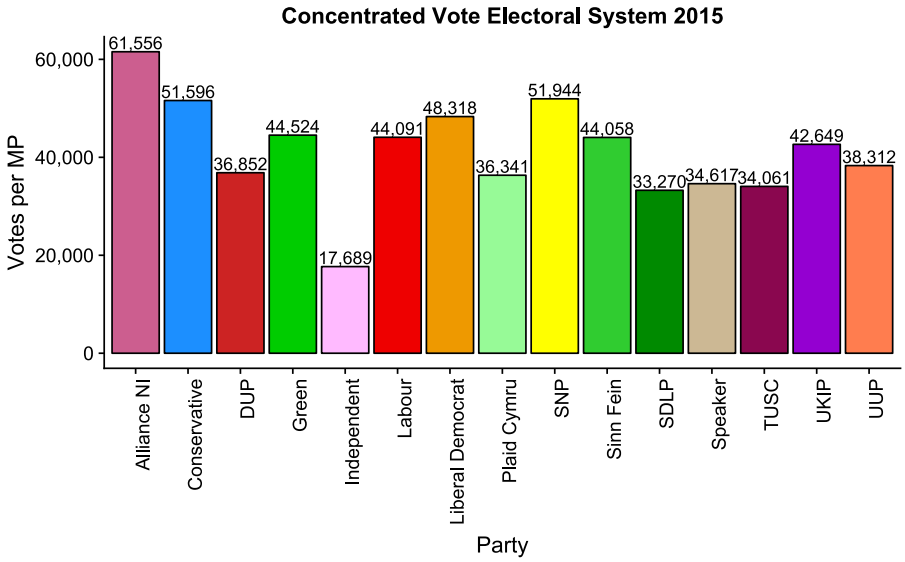
The "votes per MP elected" webpage displays the number of votes received for a party nationally for each elected MP. To calculate the votes per MP for each party, the total number of votes a party received is divided by the number of seats it won. This section of the interface gives these figures, which convey precisely how difficult or easy it is for a party to win a seat in each system. To represent this information in the interface, a bar chart is used, created with the *geom_bar* function in the ggplot2 package, with each bar representing the votes necessary for an MP to be elected in a distinct political party. The interface displays this information for both the FPTP and Concentrated Vote electoral systems.

For example, figures 3.3a and 3.3b display this information for the 2015 general election. There is a very clear difference between the results of both electoral systems. An important difference to note is the scale of the y-axis in both of the bar charts. The y-axis of figure 3.3a of the FPTP results varies between 0 and 4 million. Additionally, there is significant variation between parties as well as extreme values at both ends of this scale. However, the y-axis of figure 3.3b of the Concentrated Vote results varies only between 0 and 60,000, with minor variation between parties. To be elected in the Concentrated Vote system, MPs from all parties must have, roughly, a similar number of votes, whereas FPTP deviates greatly depending on the party. The exception to this is the votes for the Independent candidate in figure 3.3b, which has a value significantly less than the other parties. This is because only a single independent candidate in the UK was elected and it was in a relatively small constituency in Northern Ireland. Additionally, this constituency was one of the constituencies where votes did not become fully concentrated before a constraint was broken causing the algorithm to terminate.

As previously mentioned, for a party to win seats in the FPTP system it is necessary that their support is geographically concentrated. Severely disproportionate results can occur if support for a party is geographically spread. The most obvious example of this



(a) FPTP Votes per MP Elected Graphic



(b) Concentrated Vote Votes per MP Elected Graphic

Figure 3.3: Votes per MP Elected, 2015 General Election

underrepresentation was UKIP's performance in the 2015 general election, which can clearly be seen in figure 3.3a. Despite receiving 12.6% of the votes cast in the election, UKIP won just 0.2% of the seats, which is just a single seat. This meant 3.9 million votes were cast for UKIP to win a seat, whereas in comparison the Conservatives needed just 34,241. In the Concentrated Vote system, UKIP would have needed 42,649 votes and the Conservatives 51,596 votes for an MP to be elected, which would have been a fairer reflection of the levels of national support for both parties.

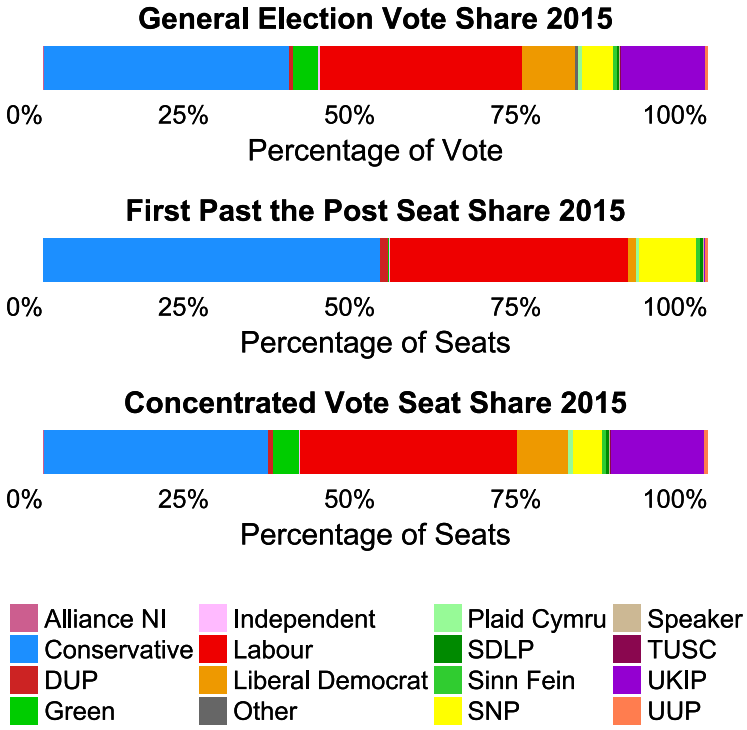
The Green party [1] and Liberal Democrats are routinely affected by this obstacle of geographically spread support. As figure 3.3a shows, the Green party needed over 1 million votes and the Liberal Democrats over 300,000 for an MP to be elected in the FPTP system. Unlike UKIP, whose popularity peaked in 2015, the Green party and the Liberal Democrats have faced this issue continuously at every general election, consistently needing significantly more votes than other parties. The Concentrated Vote allows the Green party and the Liberal Democrats to win seats with no greater difficulty than the Conservatives and Labour, as can be seen in figure 3.3b.

3.4.3.4 Vote versus Seat Share

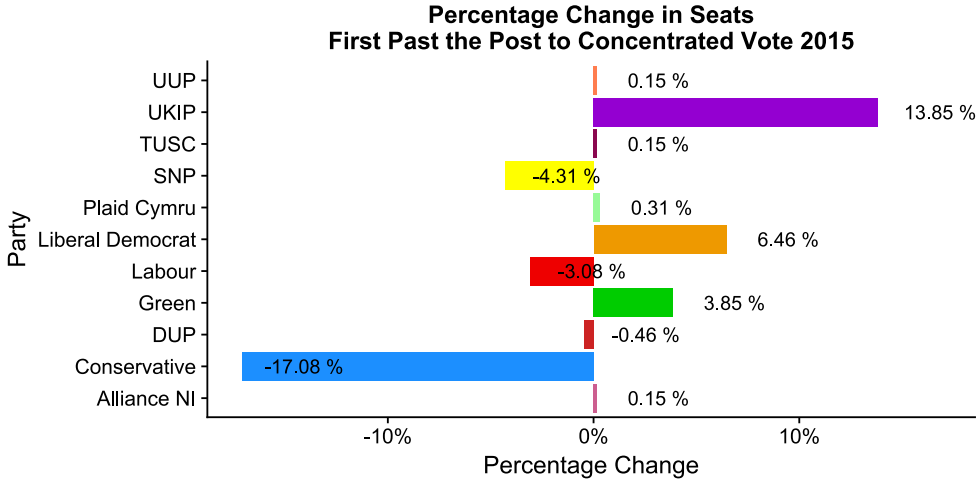
Proportional electoral systems aim to reflect the distribution of views of the electorate in those who are elected (Tideman, 1995). The vote versus seat share webpage aims to allow an evaluation of whether the Concentrated Vote system has produced results that are more in the line with how the electorate voted compared with the FPTP system. On this webpage there are two distinct visualisations: bar charts of the percentage distribution of votes and seats between parties and a chart of the percentage change in seats between the FPTP and Concentrated Vote systems.

Figure 3.4a shows the first visualisation on the webpage, which displays the makeup of the vote and seats share for both systems for the 2015 general election. The three horizontal bars represent the share of the vote each party received, the share of the seats each party achieved under the FPTP system, and the share of the seats each party would have achieved under the Concentrated Vote system, respectively. The bars were created with the *geom_bar* function in the ggplot2 package. The aim of the Concentrated Vote is to match the percentage share of the vote as closely to the percentage share of seats as closely as possible, so that the views of the whole population are taken into account, and not just those of the majority as FPTP does. Figure 3.4a shows the disproportionate relationship between the percentage of the vote and the percentage of FPTP seats achieved by the Conservatives. Despite winning only 37% of the popular vote in 2015, the Conservatives won 51% of the seats. Under the Concentrated Vote system, the Conservatives would have won 34% of the seats, which is considerably closer to their percentage share of the vote. The reverse of this occurred for many small parties, most noticeably, UKIP. Under the Concentrated Vote, UKIP would have 14% of the seats, again considerably closer their percentage share of the vote. Overall,

[1] The Green party refers collectively to the English and Welsh, Scottish, and Northern Irish Green parties.



(a) Vote versus Seat Share Graphic



(b) Percentage Change in Seats between FPTP and Concentrated Vote Graphic

Figure 3.4: Vote versus Seat Share Graphics, 2015 General Election

the bar representing the vote share is very much in line with the bar representing the Concentrated Vote seat share, and neither are in line with the bar representing the FPTP seat share.

The second visualisation, figure 3.4b, on the webpage is a bar chart showing the percentage change in seats between the FPTP and Concentrated Vote systems. The bar chart was again created with the *geom_bar* function in the ggplot2 package. As is

expected, the largest percentage changes are for the small and large parties, namely, the Conservatives, Labour, UKIP, the Liberal Democrats, and the Green Party, with the large parties losing a percentage of their seats, and small parties gaining a considerable percentage to become more equal to the percentage share of the vote.

3.4.3.5 Result Changes

The result changes webpage contains information describing the changes in seats between the FPTP and Concentrated Vote systems. There is a histogram that demonstrates where the candidates elected by the Concentrated Vote system originally ranked in their constituencies based on the FPTP results. Where a candidate was ranked first, they received the most votes in their constituency and the same candidate is elected in both systems. Where the Concentrated Vote has not elected the candidate with the most votes, the position the candidate originally ranked is recorded in the histogram and information relating to the constituency and the original votes received is displayed in a table on the web interface. Figure 3.5 shows the rank of the candidates elected by the Concentrated Vote system for the 2015 general election. The majority of the changes in seat wins are between the top three candidates in a constituency. However, there are a number of exceptional changes, where the candidate with the smallest number of votes in a constituency wins the seat. This issue is discussed further in chapter 4, section 4.3 on the limitations of the Concentrated Vote system.

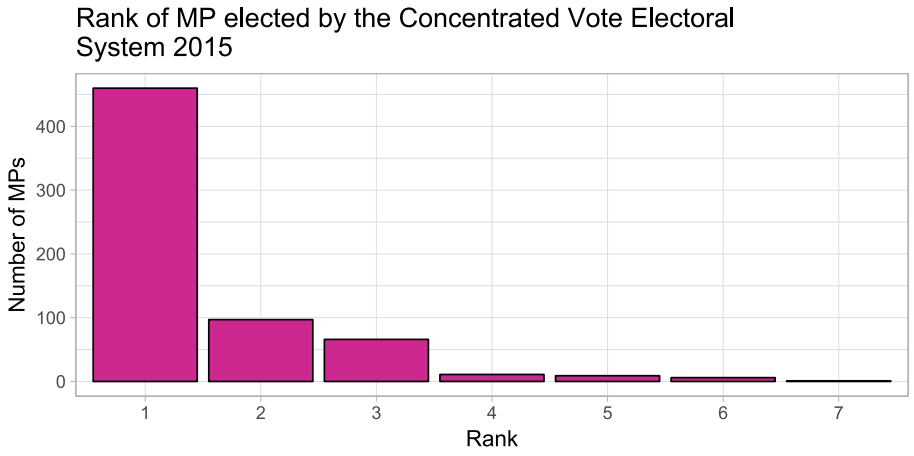


Figure 3.5: Rank of Changed Seats between FPTP and Concentrated Vote Graphic

The table that appears complementary to the histogram displays information on each constituency where the elected candidate has changed between the two systems. The table contains the winning party under FPTP and Concentrated Vote and the votes

those parties received originally in the constituency, as well as the rank of the candidate elected by Concentrated Vote. The table has a search function allowing the user to search for any value in the table, for example, a particular constituency or political party.

3.4.3.6 Proportional Maps

As detailed in section 3.4.3.1, a proportional map, or more formally a cartogram, is a map that displays geographic regions in proportion to their population or another property (Gastner and Newman, 2004). As population density is vastly different throughout the UK, displaying results on a geographic map can be misleading and for this reason, cartograms were prepared with equally sized hexagons representing each constituency, making constituencies easily comparable. The popularity of cartograms grew during the 2015 general election, with most large news reporters, such as the Independent, Sky, and the Guardian, creating their own (Field, 2015). As described in detail in section 3.4.3.1, the cartograms were prepared using identical steps. Figure 3.6 shows cartograms of the 2015 general election results for the FPTP and Concentrated Vote systems respectively. The left cartogram of the FPTP results is predominantly blue, red, and yellow representing the Conservatives, Labour, and SNP respectively. In contrast, the cartogram on the right of the Concentrated Vote results is more varied, with noticeably more purple, green, and orange representing UKIP, the Green party, and the Liberal Democrats respectively.

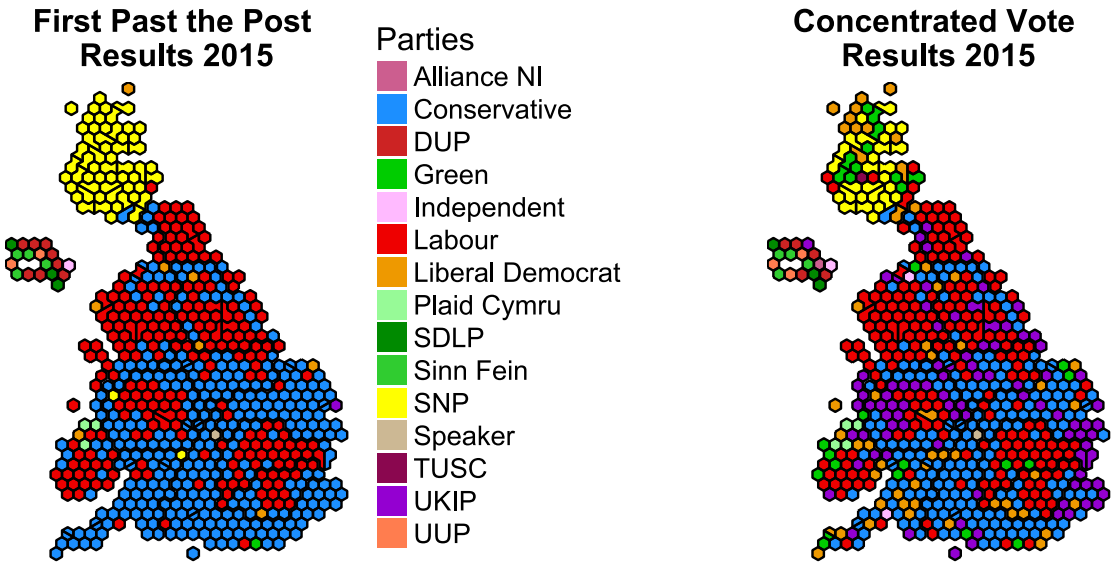


Figure 3.6: Cartograms of the General Election Results 2015

Additionally, the proportional maps webpage displays a table of information which details changes to party seats. It displays, for each party, the percentage of the vote they received, the percentage of FPTP seats won, the percentage of Concentrated Vote seats won, and the seat difference between the two systems.

3.4.3.7 Constituency Sizes

Electoral bias relating to the constituency sizes is a well known issue in the UK (Beisbart and Bovens, 2011). It is often claimed by politicians and political commentators that the current constituency boundaries are biased against the Conservatives in favour of Labour (Borisyuk et al., 2009). This is a result of the variation in constituency sizes and voter turnout. The Conservatives tend to win seats in larger constituencies while Labour tend to win seats in smaller constituencies with a lower voter turnout (Beisbart and Bovens, 2011). Following the Parliamentary Voting System and Constituencies Act 1986, the Boundary Commission, an independent body, has sought to equalise constituency sizes (Baxter, 2017).

To explore this issue further, boxplots were used to investigate the constituency sizes of Conservative and Labour seats wins under both the FPTP and Concentrated Vote system. Figure 3.7, built using the *geom_boxplot* function in the ggplot2 package, displays the boxplots of the votes cast per constituency for Conservative and Labour seat wins in the 2015 general election for both systems. A boxplot is a method of graphically representing groups of numerical data through their quartiles. Five values from the data are depicted in a boxplot: the upper and lower quartiles, represented by the top and bottom of the box, the median representing by the middle line in the box, the extreme non-outlier values, represented by the vertical lines from the box, and outliers represented by the data points outside these ranges (McGill et al., 1978).

Figure 3.7 shows there is a strong connection between Conservative seats wins and larger constituencies and Labour seat wins and smaller constituencies, as the Conservative box in the left plot sits on the y-axis noticeably higher than the box representing Labour. Interestingly, the Concentrated Vote successfully reduces this electoral bias, as can be seen in the plot on the right. Both the boxes for the Conservatives and Labour now contain roughly similar values and the size of a constituency does not appear to influence either party.

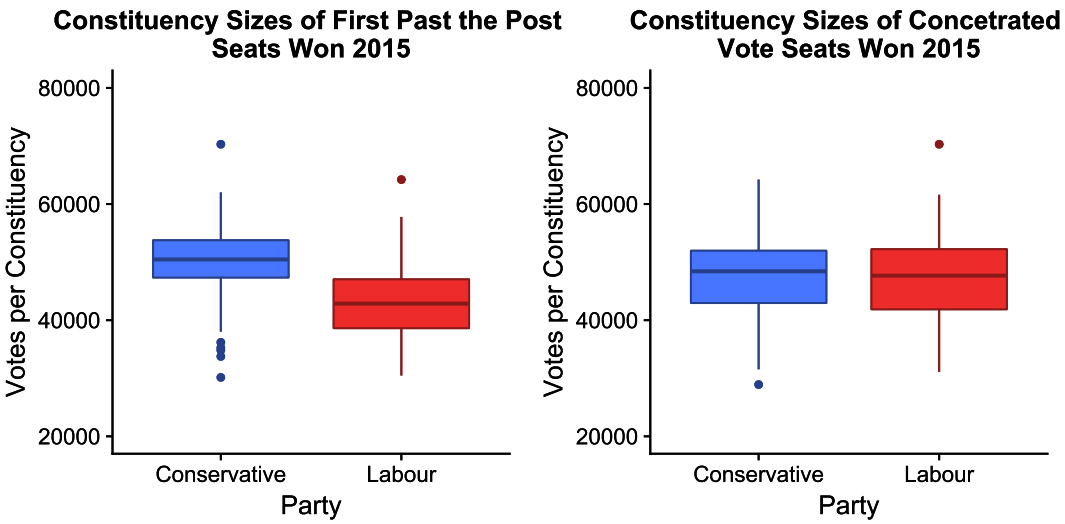


Figure 3.7: Constituency Sizes of Conservative and Labour Seat Wins

3.4.4 Interface Testing

To ensure all of the interface's features were fully functional, interface testing, also known as Graphical User Interface (GUI) testing, was employed. Interface testing verifies the interface is performing as expected and meets the necessary requirements (Memon, 2002). As the goal of the interface in this project is to be informative, rather than performing complex tasks, much of the functionality is straightforward and consequently the testing was performed manually. Test cases for the interface were drawn up to test the webpages performed as expected. A test case involves an input, execution conditions, and an expected result for a particular task (Sommerville, 2010). The testing focused on functionality of all webpages for each election year, ensuring plots were generated as expected. Additionally, the testing looked at the output on webpages containing tables of information, ensuring the search capabilities and ordering functions were executing as expected.

3.5 Summary

This chapter has detailed the methodology employed and work carried out on this project. It described the initial set up, how the data was pre-processed and the application of the scripts for the two electoral systems. The chapter then turned to the design rationale, interface design and the visualisations chosen for the interface, and described the R shiny interface containing the visualisations of the results of the application of electoral systems.

Chapter 4

Evaluation and Findings

Chapters 1 and 2 introduced the Concentrated Vote electoral system, explained the motivation underpinning the development of the new electoral system, and explored the relevant literature. Chapter 3 described the methodology used to test the hypothesis set out in section 1.3, including data preparation, application of the electoral system, and the interface design and development. This chapter evaluates and discusses both the findings of application of the Concentrated Vote to UK general election data and the web interface. The evaluation is two-fold: firstly, an evaluation of the usability and comprehensibility of the web interface and visualisations and secondly, an evaluation of the Concentrated Vote's ability to achieve a proportional distribution of votes. The limitations and strengths of the Concentrated Vote are also examined. The final chapter will then summarise the work discussed in this project and present conclusions, as well as considering possible future work.

4.1 Interface Evaluation

Interface evaluation is driven by how well the design, or aspects of the design, satisfy the user's needs (Rogers et al., 2011). To achieve the human-centred design discussed in section 3.3.1, the design needs to be evaluated by users as it is developed, and amended based on their responses (Gould and Lewis, 1985). Two approaches to interface evaluation were taken: observing users through the think-aloud technique and asking users via a questionnaire. Think-aloud was first performed with a small set of three users, incorporating each of their feedback into the design, and then a questionnaire was distributed to a group of 12 students to receive further feedback on the usability and comprehensibility of the interface.

4.1.1 Think-Aloud Technique

The first stage of the interface evaluation was performing the think-aloud technique, providing qualitative results. The think-aloud technique requires users to speak aloud while performing a task (Jaspers et al., 2004). The method was originally used in psychology research on cognitive processes and for knowledge acquisition in the context of computer systems. Think-aloud provides a unique source of information on cognitive processes allowing a direct insight into how well the user understands the interface (Jaspers et al., 2004).

Three users took part in the think-aloud evaluation which was performed in a controlled environment, the Informatics Lab in Appleton Tower at the University of Edinburgh. Each user took part individually and following each session, their comments were used to amend aspects of the interface that were found to be unclear or confusing. Short, clear instructions were given to the participants to avoid influencing their behaviour in any way. They were asked to speak aloud their thoughts as they work through the webpages in the interface and to avoid asking questions until the end.

There were two areas of confusion that were addressed by the think-aloud sessions. The first area user 1 and user 2 found particularly confusing was the “Result Changes” webpage due to a short description of the visual and an unclear explanation of what exactly was meant by the “rank” of a candidate in a constituency. Based on this feedback, a more comprehensive description of the visual was introduced. Following this, no such difficulties were found by user 3.

The second area of note from the think-aloud sessions was the user reactions to the “Wasted Votes” webpage. Prior to the evaluation, the wasted votes for FPTP and Concentrated Vote were displayed on two separate plots, side by side. This meant that the axes were on significantly different scales, as the FPTP wasted votes were materially higher than that of the Concentrated Vote. Each of the participants noted that they initially thought the results looked the same for both systems as it was not obvious how different the axes scales were. The wasted votes plot was altered so that the wasted votes for each year are displayed side by side on a single plot. This highlights precisely how different the results are.

4.1.2 Questionnaire

The second stage of the interface evaluation was a questionnaire, providing both qualitative and quantitative results. The questionnaire was prepared using Google forms and

was based on the Questionnaire for User Interaction Satisfaction (QUIS) Version 7.0 developed at the University of Maryland (Chin et al., 1988; QUIS, 2017). It is one of the most widely used questionnaires for evaluating interfaces and their design, despite being initially developed to measure user satisfaction (Rogers et al., 2011). It was chosen to evaluate the interface as it has been through many phases of refinement and has been used extensively in many evaluation studies. QUIS contains 12 sections in total:

1. System experience;
2. Past experience;
3. Overall user reactions;
4. Screen design;
5. Terminology and system information;
6. Learning;
7. System capabilities;
8. Technical manuals and online help;
9. Online tutorials;
10. Multimedia;
11. Teleconferencing; and
12. Software installation (Rogers et al., 2011).

For the purposes of the questionnaire for this project, sections 3, 4, 6, and 7 were the most important aspects to evaluate. Each section contains a number of related questions and the most relevant questions were selected for inclusion in the questionnaire. Almost all of the questions are set on semantic differential scales between 1 and 9, which explore a range of bipolar attitudes to a particular item (Rogers et al., 2011). The participant is asked to rate their position between two extremes, for example between terrible, being 1, and wonderful, being 9. A single question at the end of each section was an open question, which asked for the participant's thoughts and comments of the section. Open questions were kept to a minimum as they typically receive a lower response rate and produce more missing data than closed questions (Reja et al., 2003). The full questionnaire is included in appendix C .

The questionnaire was distributed, following adjustments to the interface based on the user feedback from the think-aloud sessions discussed in section 4.1.1, to a group of 12 students. The results of each of the four sections of the questionnaire are discussed in more detail below.

4.1.2.1 Overall User Reactions

The overall user reaction section aimed to gauge the user's general impression of the interface. The questionnaire asked 7 questions in relation to the user's overall reaction to the interface, 6 of which were set on semantic differential scales and 1 of which was an open question on the user's thoughts or comments. The results from this section were positive. The first question asked the participants to rate their overall reaction to the interface between terrible, being 1, and wonderful, being 9. Of the respondents, 92% rated their reaction as 7 or above. Similarly, the participants were asked to rate their overall reaction to the interface between difficult, being 1, and easy, being 9. 100% of the respondents rated the interface as 7 or above, with 50% selecting 9. The responses to the remaining questions in this section were of a similar pattern, with respondents rating between 4 and 9, with the majority of responses between 7 and 9. The results for all questions are included in appendix D.

4.1.2.2 Screen Design

The objective of the screen design section was to understand whether the interface design was helpful to the user's understanding of the visualisations of the data displayed. The questions were focused on the screen layout, the amount of information displayed and its organisation, and the sequence of the screens. Respondents were asked to rate if the screen layouts were helpful between never, being 1, and always, being 9. All respondents rated 6 or above, with 50% rating 9. Likewise, respondents rated 6 or above when asked about the amount of information displayed and its organisation. The results of all questions are included in appendix D.

4.1.2.3 Learning

The aim of the learning section was to understand how easy the respondents found the interface to use and to understand the information displayed. The questions concentrated on learning to operate the interface, exploring features, and performing tasks. The feedback from each of the questions in this section was very positive, with all questions receiving responses of 7 or higher. There were no issues reported with learning to use the system or performing tasks. All respondents rated 8 or 9 when asked if performing tasks were straightforward on a scale between never, being 1, and always, being 9. The results of all questions are included in appendix D.

4.1.2.4 System Capabilities

The final section of the questionnaire addressed the system's capabilities, including system speed, operation response times, and the levels of user the system was designed for. The participants were asked to rate the system speed between too slow, being 1, and fast enough, being 9. Respondents rated between 4 and 9, with over 50% rating 8 or higher. A similar response was obtained for the question on operation response times. As the *shinyapps.io* server is used, which is a subscription service, a higher level account type would need to be purchased to improve these aspects of the interface's capabilities. However, as the responses were not particularly negative, the expense of a different subscription type was deemed not to be necessary. The participants were also asked to rate between never, being 1, and always, being 9, if they thought the system was designed for all levels of users. All respondents rated 6 or above, with 67% rating 8 or 9. See appendix D for the results of all questions.

4.2 Concentrated Vote Electoral System Evaluation

The hypothesis of the Concentrated Vote electoral system is that it improves proportionality in election results, compared with the FPTP system. In order to evaluate this hypothesis, the differences in proportionality between the results of the two systems must be measured. This measurement can be done in a number of ways and the following criteria were used to evaluate the systems:

- Political science disproportionality indices;
- Comparison with alternative system results; and
- Other proportionality measures.

Each of these items will be discussed in detail below.

4.2.1 Measuring Disproportionality

Proportional electoral systems aim to minimise disproportionality by producing results where the proportion of seats won per party is as close as possible to their share of the vote (Gallagher, 1991). In political science, measuring disproprotionality is a difficult task because, depending on what is meant by "disproprotionality" and the method used to calculate it, results can vary (Renwick, 2015a). Although there is no standardised definition of what exactly is disproportionality or a method by which it should be

measured, there are a number of well-established indices for this purpose, including the Loosemore-Hanby index, the Gallagher index (both absolute measures), and the Sainte-Laguë index (a relative measure). These indices were applied to the results in each election year of the FPTP and Concentrated Vote systems to measure whether there was a decrease in disproportionality of the results with Concentrated Vote compared to the FPTP results, tying in with the hypothesis of Concentrated Vote producing more proportional results than FPTP.

4.2.1.1 Loosemore-Hanby Index

The Loosemore-Hanby (LH) index, sometimes known as the deviation from proportionality score, is a longstanding metric in political science for measuring disproportionality in electoral systems proposed by Loosemore and Hanby (1971). The index is the total percentage by which the overrepresented parties are overrepresented (Lijphart and Aitkin, 1994). It is calculated by first adding the absolute values of all vote-seat share differences and then dividing this value by 2. This is displayed in equation 4.1, with v, being the percentage of votes and s, being the percentage of seats won by party i (Loosemore and Hanby, 1971). A problem that has been noted with the LH index is that it can exaggerate disproportionality of systems with a large number of parties (Lijphart and Aitkin, 1994).

$$LH\ index = \frac{1}{2}\sum_{i=1}^{n} |v_i - s_i| \tag{4.1}$$

The LH index was calculated in R using the above formula for both the FPTP and Concentrated Vote systems. The results are displayed in table 4.1. To interpret these values, the index measures the deviation between a party's share of the vote and the share of the seats. For example, the LH index for the FPTP 2015 general election was 23.9, meaning the parties whose seat share exceeded their vote share hold 23.9% more seats than they would if a more proportional system was used. A decrease in this index indicates an improvement in the proportionality. From table 4.1, there is a clear improvement in the proportionality of the results from FPTP to the Concentrated Vote system. The LH index for each election year decreased by at least 50%, with the majority decreasing by more than 70%. This indicates that, in line with the hypothesis, the Concentrated Vote has produced results that are materially more proportional than FPTP.

LH Index		
Year	First Past the Post	Concentrated Vote
2017	10.3	3.3
2015	23.9	4.5
2010	22.8	4.4
2005	20.7	5.4
2001	22.1	5.3
1997	21.2	4.3
1992	18.0	5.2
1987	20.8	4.2
1983	23.9	3.4
1979	15.3	6.1

Table 4.1: LH Index Results for Electoral Years 1979 to 2017

4.2.1.2 Gallagher Index

The Gallagher index, also known as the least-squares index, is one of the most widely used indices in political science for measuring disproportionality in electoral systems (Renwick, 2015a) proposed by Gallagher (1991). The index gained much of its popularity following an endorsement by Lijphart and Aitkin (1994). The Gallagher index addresses the problem with the LH index discussed in section 4.2.1.1. The notable advantage over the LH index is that it registers a few large deviations more strongly than many small ones (Lijphart and Aitkin, 1994). It is calculated by first adding the squared vote-seat share differences for each party, then dividing this total by 2, and finally the square root of this value is taken (Lijphart and Aitkin, 1994). This is displayed in equation 4.2, with v, being the percentage of votes and s, being the percentage of seats won by party i (Gallagher, 1991).

$$Gallagher\ index = \sqrt{\frac{1}{2}\sum_{i=1}^{n}(v_i - s_i)^2} \tag{4.2}$$

The Gallagher index was calculated in R using the above formula for both the FPTP and Concentrated Vote systems. The results are displayed in table 4.2. The results can be interpreted in the same way as the LH index, with a lower value representing greater proportionality. The Gallagher index is the preferred measure by political scientists, primarily because it does not give the same weight to a large number of small vote-seat share differences as it does to a small number of large vote-seat share differences (Renwick, 2015a). However, this means it does not capture the grossly disproportionate results for the Liberal Democrats in 2010 and UKIP in 2015, as the values for these

years are much the same as previous years, despite these notable disproportionalities.

The results of the Gallagher index for the Concentrated Vote, compared with the FPTP results in table 4.2, provide additional confirmation that the system has improved the proportionality of the results. The index for each election year has decreased by a remarkable amount, further providing evidence to support the hypothesis.

Gallagher Index		
Year	First Past the Post	Concentrated Vote
2017	6.6	2.4
2015	15.0	2.9
2010	15.1	2.6
2005	16.6	3.5
2001	17.7	3.5
1997	16.5	2.6
1992	13.5	4.1
1987	14.9	3.3
1983	17.2	2.5
1979	11.6	4.3

Table 4.2: Gallagher Index Results for Electoral Years 1979 to 2017

4.2.1.3 Sainte-Laguë Index

A less commonly used index is the Sainte-Laguë. The Sainte-Laguë metric looks at the relative deviation, that is the deviation in proportion to a party's support (Renwick, 2015a). The first person to look at Sainte-Laguë as a relative measure of disproportionality was Gallagher (1991). The index is calculated by firstly, taking the square of the vote-seat share difference for each party, then dividing it by that party's vote share, and finally, summing these values across all parties (Goldenberg and Fisher, 2017). This is displayed in equation 4.3, with v, being the percentage of votes and s, being the percentage of seats won by party i. The major difference between the Sainte-Laguë index and the previous two indices is that it is not concerned with the absolute value of the party's share of the seats and votes, but with the relative difference (Gallagher, 1991). To demonstrate this point, Renwick (2015b) uses an example of one party receiving 30% of the vote and 25% of the seats, and another party receiving 5.1% of the vote and 0.1% of the seats. The LH and Gallagher indices see the same level of disproportionality in both situations as they look at the absolute difference. As the Sainte-Laguë index looks at relative differences, it captures the intuition that the situations are not the same and should be treated differently, thus avoiding the issue.

$$Sainte\text{-}Laguë\ index = \sum_{i=1}^{n} \frac{(s_i - v_i)^2}{v_i} \tag{4.3}$$

Although it was the index favoured by Gallagher (1991), it not widely used and fails on certain criteria discussed by Taagepera and Grofman (2003), who set out a comprehensive axiom-based review of 19 disproportionality indices. They found that the Sainte-Laguë index came in the top 4, however, failed three of the criteria. Renwick (2015a) later argued that the criteria the index falls short on may be of limited relevance and the index is at least as good as the main alternatives, the LH index and the Gallagher index. Goldenberg and Fisher (2017) have further discussed the Sainte-Laguë index and its possibility of being a standard measure of disproportionality in electoral studies. The discussion of the merits of the index continues in the literature, with no definitive conclusions reached to date.

The Sainte-Laguë index was calculated in R using the above formula for both the FPTP and Concentrated Vote systems. The results are displayed in table 4.3. As can be seen from the table, the index captures the disproportionality in 1983, where the SDP-Liberal Alliance received just 3.5% of the seats but 25.4% of vote, in 2010, where the Liberal Democrats received 9.5% of the seats and 22.1% of the vote, and in 2015, where UKIP received 0.2% of the seats and 12.6% of the vote, to a much greater extent that the two aforementioned indices. The Sainte-Laguë index for the Concentrated Vote is also significantly reduced compared to the FPTP index results in table 4.3. This indicates that, not only has the Concentrated Vote improved proportionality overall, but it has also improved the proportionality relative to each party.

Sainte-Laguë Index		
Year	First Past the Post	Concentrated Vote
2017	12.4	1.3
2015	32.7	1.7
2010	25.1	2.3
2005	25.6	2.5
2001	23.6	2.2
1997	22.1	1.9
1992	19.7	2.0
1987	24.8	1.5
1983	30.07	1.5
1979	16.6	3.2

Table 4.3: Sainte-Laguë Index Results for Electoral Years 1979 to 2017

4.2.2 Comparison with Alternative Systems

The second method of evaluating proportionality was to compare the results of FPTP and Concentrated Vote with alternative voting systems. The Electoral Reform Society, an independent organisation promoting electoral reform, commissioned a post-election survey as part of their analysis of the 2015 general election. As part of the survey, 40,000 people were asked to indicate how they would have voted if they had to rank parties in order of preference for the 2015 general election (Garland and Terry, 2015). This information was then used to estimate the results if proportional systems had been used. The proportional systems employed for the projections involve Alternative Vote (AV), party lists, and single transferable vote (STV). AV is a majoritarian system where voters rank candidates in order of preference and to win, a candidate must secure an absolute majority of the available vote. Party list systems involve voters being given a list of candidates and seats are assigned to candidates based on their party's share of the vote. STV is a proportional voting system that uses ranked votes in multi-seat constituencies (Norris, 1997). Full details of each of these systems are available in chapter 2. Given the vast size of the collected data, the results of the projections provide high-quality estimates of how the results would have looked if these systems had been employed. For this reason, these projected results will be compared against the FPTP and Concentrated Vote results for the 2015 general election to evaluate proportionality comparatively. The projected estimates do not contain data for Northern Ireland, so Northern Ireland has also been excluded from the FPTP and Concentrated Vote results discussed in this section.

Table 4.4 shows the number of seats allocated to each of the parties by each of the electoral systems. From the table, the similarities between the FPTP and AV results are very clear. As they are both non-proportional systems, more seats tend to be won by larger parties and fewer by smaller parties. The results of the Concentrated Vote, party list, and STV systems produce similar results, with STV tending to allocate slightly more seats to large parties and fewer seats to small parties compared to the two former systems.

The most significant part of evaluating the proportionality of the results is to compare the percentage of the seats won by a party to their percentage of the vote. Table 4.5 displays the share of the vote per party, as well as the share of the seats won by each party for each of the electoral systems. Of the electoral systems discussed, party lists achieve the most proportional result, with the seat share for each party being within

Seats in Alternative Systems, General Election 2015					
Party	FPTP	Concentrated Vote	Party List	AV	STV
Conservative	331	219	242	337	276
Labour	232	212	208	227	236
Liberal Democrat	8	50	47	9	26
UKIP	1	90	80	1	54
Green Party	1	26	20	1	3
SNP	56	28	30	54	34
Plaid Cymru	3	5	5	3	3

Table 4.4: Seats in Alternative Systems (Garland and Terry, 2015)

2% of the vote share. However, party lists is closely followed by Concentrated Vote, which achieves a share of seats within 3% of the vote share. The results produced by STV, although significantly more proportional than FPTP and AV, were inclined to allocate larger parties, such as the Conservatives and Labour, a larger share of the seats than the share of the vote they received, and the reverse occurred with smaller parties. As expected, FPTP and AV allocated significantly more seats to larger parties and significantly fewer to smaller parties. Both FPTP and AV allocated approximately 14% more seats to the Conservatives than they would have received based on the vote share. Conversely, FPTP and AV allocated approximately 12% fewer seats to UKIP than they would have received based on the vote share.

Percentage of Seats in Alternative Systems, General Election 2015						
Party	Vote %	FPTP Seat %	Concentrated Vote Seat %	Party List Seat %	AV Seat %	STV Seat %
Conservative	36.9%	50.9%	34.7%	37.2%	51.8%	42.5%
Labour	30.4%	35.7%	33.5%	32%	34.9%	36.3%
Liberal Democrat	7.9%	1.2%	7.9%	7.2%	1.3%	4%
UKIP	12.9%	0.2%	14.2%	12.3%	0.2%	8.3%
Green Party	3.8%	0.2%	4.1%	3.1%	0.2%	0.5%
SNP	4.7%	8.6%	4.4%	4.6%	8.3%	5.2%
Plaid Cymru	0.6%	0.5%	0.8%	0.8%	0.5%	0.5%

Table 4.5: Percentage of Seats in Alternative Systems (Garland and Terry, 2015)

Overall, the performance of the Concentrated Vote in terms of proportionality is strong compared with projections for the other proportional electoral systems, namely AV, party lists, and STV. Table 4.6 displays the Gallagher index, calculated in terms of the information available for the alternative systems, for each of the electoral systems

discussed. It shows that the results of the Concentrated Vote are as proportional, or more proportional, than the other systems. The seat share of the Concentrated Vote results is also closely aligned with the share of the vote each party received, again in line with the hypothesis.

FPTP	Concentrated Vote	Party List	AV	STV
15.1	2.9	1.4	15.4	7.5

Table 4.6: Gallagher Index of Alternative Systems, 2015 General Election

4.2.3 Other Proportionality Measures

In addition to the political science indices and comparisons discussed above, there are a number of further measures that can be employed to evaluate if the parliamentary results produced by the Concentrated Vote system are more proportional than the FPTP results. Proportional electoral systems have a number of key properties that should be present in the Concentrated Vote results, if they are in fact proportional. These properties include having a small percentage of votes wasted, similar difficulty for all parties to elect a candidate, and seats are typically not won by very small margins. Each of these measures is discussed below.

4.2.3.1 Wasted Votes

A clear difference between non-proportional and proportional systems is that non-proportional systems consider the views of the majority only and disregard the rest, whereas proportional systems aim to reflect the diversity of views among the electorate as closely as possible to that of the elected candidates. As non-proportional systems only take into account the views of the majority, a significant proportion of votes are wasted. As outlined in section 3.4.3.2, in FPTP wasted votes are votes for a losing candidate or surplus votes to the winning candidate (Garland and Terry, 2015) and in Concentrated Vote, wasted votes are those left over when the process of elimination cannot continue without breaking one of the constraints relating to the number of votes per constituency and votes per party.

Table 4.7 presents the number and percentage of wasted votes for FPTP and Concentrated Vote for the electoral years 1979 to 2017. If the hypothesis holds, the number of wasted votes should be dramatically reduced by Concentrated Vote. The results in

table 4.7 show that this is very clearly the case. In 2015, there were 23 million votes wasted by FPTP, or 74% of all votes cast, whereas Concentrated Vote wasted 234,334, which is just 0.76% of votes cast. The results for the other election years are similar, with a sharp decrease in the number of wasted votes with Concentrated Vote, as shown in table 4.7.

Details of Wasted Votes				
Year	FPTP Wasted Votes	Concentrated Vote Wasted Votes	FPTP % Wasted	Concentrated Vote % Wasted
2017	22,016,215	165,516	68.42%	0.51%
2015	22,813,937	234,334	74.33%	0.76%
2010	21,122,519	287,814	71.15%	0.97%
2005	19,159,430	263,684	70.75%	0.97%
2001	18,573,027	212,848	70.45%	0.81%
1997	22,219,761	221,747	71.03%	0.71%
1992	23,099,145	189,059	68.72%	0.56%
1987	22,690,160	150,629	69.75%	0.46%
1983	21,297,953	282,310	69.63%	0.92%
1979	21,222,055	254,513	67.96%	0.82%

Table 4.7: Wasted Votes for Electoral Years 1979 to 2017

4.2.3.2 Votes per MP Elected

The number of votes required for an MP to be elected in each party is a metric that can easily demonstrate proportionality. If the hypothesis holds and the Concentrated Vote system is more proportional, the number of votes for a candidate to be elected as an MP should be roughly the same per party. If the number of votes per MP elected varies widely between political parties, this suggests a high level of disproportionality which regularly occurs with FPTP. Table 4.8 shows the number of votes required for an MP to be elected in both the FPTP and Concentrated Vote systems. Where a row is blank in the FPTP column of the table, this means that no candidate from that particular party was elected under FPTP.

Table 4.8 starkly highlights the disproportionality of FPTP and proportionality of Concentrated Vote. FPTP requires smaller parties, such as UKIP and the Green Party, to get millions of votes for just a single seat, whereas larger parties like the Conservatives require just 34,000. In comparison, Concentrated Vote requires parties to get a roughly similar number of votes to be elected, generally between 30,000 and 40,000 votes. The

results for remaining election years between 1979 and 2017 follow the same pattern, and are available to view in appendix E.

Votes per MP Elected, General Election 2015		
Party	First Past the Post	Concentrated Vote
Alliance NI	-	61,556
Conservative	34,241	51,596
DUP	23,032	36,853
Green Party	1,157,630	44,524
Independent	98,481	17,689
Labour	40,290	44,091
Liberal Democrat	301,990	48,318
Plaid Cymru	60,568	36,341
SNP	25,972	51,944
Sinn Fein	44,058	33,270
SDLP	33,270	33,270
Speaker	34,617	34,617
UKIP	3,881,099	42,649
TUSC	-	34,061
UUP	57,468	38,312

Table 4.8: Votes per MP Elected for the 2015 General Election

4.2.3.3 Marginal Seats

Another method to evaluate whether Concentrated Vote has improved the proportionality of the results is to analyse the seats won by a marginal percentage in the FPTP system. If the hypothesis of increased proportionality holds, these seats should switch under Concentrated Vote and the seat should no longer be a marginal win for the elected candidate.

Table 4.9 shows the details of a selection of constituencies where the candidate with the most votes, elected by FPTP, was within 10% of the candidate with the next highest number of votes in the 2015 general election. Details of all constituencies in this category are included in appendix F. Every constituency that fell within this category had a different candidate elected by the Concentrated Vote system, typically the winning party switching from Conservative to Labour or Liberal Democrat. Although the votes cast are very similar for the FPTP and Concentrated Vote winners, the votes for each winner are very different after the Concentrated Vote algorithm has been run. The rightmost column displays the votes the Concentrated Vote winner had when the algorithm terminated. For each of the constituencies in table 4.9, they were the

only candidate in their constituency left standing. This is because, as described in section 2.3, the Concentrated Vote shifts votes between constituencies to where a party started out strong and ends up with constituencies where votes for a party have become concentrated.

FPTP Marginal Seat Win Changes in Concentrated Vote					
Constituency	First Past the Post Winner	FPTP Votes	Concentrated Vote Winner	FPTP Votes	CV Votes
Bury North	Conservative	18,970	Labour	18,592	45,230
Cambridge	Labour	18,646	Liberal Democrat	18,047	51,587
Croydon Central	Conservative	22,753	Labour	22,588	52,884
Eastbourne	Conservative	20,934	Liberal Democrat	20,201	52,768
Gower	Conservative	15,862	Labour	15,835	42,337
Morley and Outwood	Conservative	18,776	Labour	18,354	47,771
Thornbury and Yate	Conservative	19,924	Liberal Democrat	18,429	48,570
Thurrock	Conservative	16,692	UKIP	15,718	49,454
Weaver Vale	Conservative	20,227	Labour	19,421	46,867
Ynys Mon	Labour	10,871	Plaid Cymru	10,642	34,778

Table 4.9: Marginal Seats Changes (less than 10%), 2015 General Election

The results of this analysis further support the hypothesis of increased proportionality with Concentrated Vote. The seats that were won in the FPTP system by a marginal percentage do not remain as wins in the Concentrated Vote system as the votes of other parties in the constituency are taken into account. For example, in the results for Gower in table 4.9, the Conservatives won the seat over Labour by just 27 votes under FPTP, whereas Labour wins with Concentrated Vote. This is an extreme case, but there is clearly a pattern of this type of win from the information in the table. The FPTP system would have excluded these votes for losing candidates, despite them making up the views of a considerable proportion of the constituency. The Concentrated Vote includes these votes while allocating seats and because of this, allocates parliamentary seats much more closely to the views of the electorate.

4.3 Limitations of Concentrated Vote

The information presented for the hypothesis in section 4.2 above provides strong evidence that Concentrated Vote produces proportional results, particularly compared to FPTP. However, there are a number of limitations of Concentrated Vote as an electoral system to be considered. The most substantial limitation to the implementation of this

system is public acceptance, especially in light of the response to Alternative Vote during the 2011 referendum on electoral reform. Other possible limitations include the impact of shifting votes between constituencies, the speed at which the system takes to run, and changes to voting behaviour.

4.3.1 Public Acceptance

The support for Concentrated Vote by both the public and political parties is a significant obstacle to the use of Concentrated Vote as the electoral system in the UK. The complex mathematical nature of the system could be difficult for voters to comprehend, and a difficult "sell" in a referendum campaign. Additionally, larger political parties, which are the main beneficiaries of the disproportionality of FPTP, favour the retention of FPTP and are unlikely to change tack to campaign for a system that would erode their number of seats in parliament.

At a high level, the premise of Concentrated Vote is straightforward; votes for candidates are concentrated based on where support for a particular party is strongest. However, two aspects of the process are a cause for concern in the public arena. Firstly, it is crucial that the public understand that number of votes in the process is fixed and that votes are simply transferred between constituencies, not duplicated or multiplied upwards in any way. A misunderstanding about the transfers would be detrimental to public support for Concentrated Vote. Secondly, the mathematical details of how exactly votes are concentrated are complex, and not easily understood. As any change to the UK's electoral system requires ratification by referendum, any potential obstacle to public understanding of the system could mean they are unlikely to support it. The involvement of complicated mathematics in the process is easily criticised as deliberately obscuring the process or tampering with the public will in some way. Unless the public have a clear understanding of why votes are concentrated and the benefits of allocating *transferred* votes in this way, it is extremely difficult to refute these arguments in a way that will resonate with the public.

As detailed in section 2.4.2, the 2011 referendum on replacing FPTP with Alternative Vote (AV) was not a success. FPTP has long been used in the UK and is especially easy for the public to understand, both in its own right and compared to the other electoral systems discussed in this project. One reason, among many, that the 2011 referendum was unsuccessful was the complicated explanation of AV provided by the Electoral Commission. Given the simple nature of AV and the relatively poor reception it received

from the public, the acceptance of an electoral system with a strong mathematical foundation appears unlikely.

Support from political parties is another issue. The large UK parties, the Conservatives and Labour, have large bases of support and a strong ability to influence the public, both in terms of their party membership and ability to mobilise campaigners on the ground. As FPTP favours large parties, it is unlikely these parties, would support a proportional electoral system likely to erode their number of seats or even cost them a majority in a general election. This issue also contributed to the rejection of AV in the 2011 referendum. The Conservatives took a strong public position against AV and many Labour politicians spoke publicly against it, which undoubtedly played a role in the formation of a public view on AV.

4.3.2 Constituency Link

A common criticism of proportional representation (PR) is that it breaks the link between a constituency and the candidate elected to represent it (Electoral Reform Society, 2015), a link which FPTP is often praised for maintaining. The reason for the maintenance of the constituency link with FPTP is because PR systems typically elect more than a single representative per constituency, whereas FPTP just elects one. Although Concentrated Vote maintains a single representative per constituency, the votes in a constituency can be transferred to another constituency, which could frustrate supporters of FPTP. With candidates elected in constituencies with just 24.5% of the vote, however, as was the case in South Belfast for an SDLP candidate in the 2015 election, it can be easily argued that there is no such link with the constituency to begin with. Over half of the elected MPs in 2015 did not have support from the majority of their respective constituencies. Furthermore, a survey in 2013 revealed that only 22% of people knew who their local MP was (Electoral Reform Society, 2015).

There is concern regarding some of the candidates the Concentrated Vote causes to be elected. Although overall the results produced are proportional, there are occasionally constituencies that end up with the candidate with the least number of votes in that constituency being elected. Adjustments to the systems would be required to prevent this from happening, as it would be extremely unlikely that a constituency would support such an election.

4.3.3 System Runtime

A possible limitation of Concentrated Vote is the time taken for the system to run. The first stage of Concentrated Vote is votes being cast and counted in the same format as FPTP. This is a relatively quick process completed manually by count teams who count and record the number of votes for each candidate. To apply the final step of Concentrated Vote, which is applying the algorithm in R discussed in section 3.2.3, the results for all constituencies are required. As constituencies with close results between candidates often require multiple recounts of votes, this could be a potential delay to the entire process. However, as these marginal differences do not impact seat wins to the same extent in Concentrated Vote as they do in FPTP, the emphasis given to recounts may be lessened.

The application of the algorithm in R takes approximately 10 minutes to run. However, should the Concentrated Vote be run to calculate the results of a real election, further tests and checks would need to be employed to ensure the correctness of the results. Although the system runtime is longer than that of FPTP, it is comparable to proportional systems, such as STV, where there is a lengthy counting process with redistributing results and quota calculations.

4.3.4 Voting Behaviour

A possible limitation of the results obtained in this project for the Concentrated Vote is the electoral data. It represents results where voters planned to vote under FPTP, with knowledge of all the flaws of that system. While one would think the electorate would likely vote in a substantially similar way under Concentrated Vote, as it is also voting for a single representative per constituency, the extent to which people's voting behaviour may change if they knew there was a possibility for their vote to be shifted to another constituency is not known. For example, would more people vote for the Liberal Democrats in strong Conservative or Labour constituencies if they knew the vote could help the Liberal Democrats in another part of the country? Or would the electorate be put off voting for alternative options, like UKIP, if they thought their vote would count? The extent to which considerations such as these affects the available data is unknown and there is no way to measure its potential impact. However, it is a limitation of the results in this report worth noting.

4.4 Strengths of Concentrated Vote

Despite the limitations noted in section 4.3, there are a number of significant strengths of the Concentrated Vote system. The major strength of the system is its ability to translate the views of the electorate to parliamentary seats that accurately reflect these views. Other strengths include a reduction in wasted votes and the inclusion of minority voices. Each of these is discussed in more detail below.

4.4.1 Representative Results

The key strength of the Concentrated Vote is that it has been shown to produce results where the distribution of views among those who are elected is representative of the distribution of views of the electorate. This is something that is not achievable with FPTP as such a large proportion of votes are not taken into consideration. Proportional systems almost always result in coalition governments (Reynolds et al., 2008), which can be seen as a strength as this means more voters can be represented by a single government.

4.4.2 Simplicity of Casting Votes

A key benefit of FPTP is its simplicity, voters need only select a single candidate on their ballot paper. This aspect of FPTP has been maintained by Concentrated Vote. Votes will still be cast in the same format and a single candidate elected in each constituency.

4.4.3 Inclusion of Minority Voices

The inclusion of minority voices and independent candidates is a noteworthy strength of Concentrated Vote. Where FPTP makes it difficult for candidates from smaller parties or parties with evenly spread support to win seats, Concentrated Vote takes these views into account.

4.4.4 Reduction in Wasted Votes

As discussed in section 3.4.3.2, the number of wasted votes are dramatically reduced by Concentrated Vote and following from this, votes in “safe seat” constituencies are taken into account. Wasted votes are reduced as votes for candidates, other than the winning candidate, are transferred to constituencies where their party started out stronger. From

this, votes in safe seat constituencies for candidates, other than the winning candidate, will be taken into account.

4.5 Summary

This chapter has set out the methods used to evaluate both the web interface and the proportionality of the Concentrated Vote results. It outlined the results of these evaluations, namely that the web interface was well designed and Concentrated Vote achieved more proportional results than FPTP. The limitations and strengths of Concentrated Vote were also discussed.

Chapter 5

Conclusion

This report has set out the issues with the UK's current electoral system, FPTP, which led to the development of the Concentrated Vote electoral system. This project had the hypothesis that Concentrated Vote would increase proportionality if it were used as the UK's electoral system. To test the hypothesis of increased proportionality, this project applied Concentrated Vote to UK general election data from all general elections between 1979 and 2017. The project also sought to create visualisations of these election results for use on a specifically designed web interface. This chapter provides a summary of the findings of this project and discusses possible further work.

5.1 Summary of Findings

The findings of this project present evidence in favour of the hypothesis stated in section 1.3. The hypothesis of increased proportionality was measured using several evaluation methods, which were set out in chapter 4. The evaluation methods included applying three commonly used disproportionality indices in political science: the Loosemore-Hanby, Gallagher, and Sainte-Laguë indices, as well as comparing the results with projections of alternative systems for the 2015 general election results, and finally, other measures such as the percentage of wasted votes, votes per MP elected, and changes to marginal seat wins, were also analysed. The results produced by Concentrated Vote for the election years from 1979 to 2017 were all found to improve proportionality based on each of the evaluation techniques employed.

A web interface was developed with a variety of visualisations to present the results of the application of Concentrated Vote to general election data and to inform users of the Concentrated Vote electoral system. The web interface aimed to be easily used,

comprehensible, and engaging for users on the topic of proportionality of election results. The visualisations included on the web interface were of the FPTP and Concentrated Vote electoral system results for the years 1979 to 2017 and included webpages dedicated to topics including wasted votes, votes per MP elected, proportional maps, and changes between the two systems' results. The usability and comprehensibility of the interface were tested using the think-aloud technique and a questionnaire. The vast majority of the user feedback indicated the interface was easy to use and understand and the information provided was stimulating and engaging.

The Concentrated Vote system is not without limitations. The first and most critical being the public perception of this highly mathematically system. Any difficulties understanding how the system works could result in a negative public view, which would impede its possible adoption. The second limitation is the constituency link with an MP. A vote being transferred to another constituency may not sit well with the public, as well as the possibility of a candidate with the least number of votes in a constituency being elected. The final limitation is the speed at which the system can be run. An advantage of FPTP is the speed at which votes can be counted and the winning candidate announced. Although the time taken to count and prepare the results would be increased by Concentrated Vote, it would not be a significantly greater amount of time, particularly compared with other proportional systems. Nevertheless, the system's key strengths - the results produced being proportionate between the votes cast and the seats won per party as well as maintaining the simplicity of casting votes from FPTP - could potentially outweigh these limitations.

5.2 Further Work

Each of the limitations of Concentrated Vote, discussed in section 4.3, are areas of possible further work. The possibility of a candidate with the smallest number of votes in a constituency being elected is a particularly important issue. Research into ensuring the candidate elected in a constituency is not unpopular in its respective constituency is key to ensuring public acceptance. However, modifying the algorithm to take account of this issue, while preserving the other metrics of concentrating votes could prove to be difficult mathematically.

As discussed in section 3.2.3, the data for Northern Ireland and the rest of the UK was separated due to there being distinct differences in the parties standing in these areas. Another possible area of further work is applying the Concentrated Vote on a

country or regional level.

Additionally, further research could be completed to gauge the public's reaction to the Concentrated Vote system. As part of this project, the web interface developed could be used as a tool to introduce participants to the system. Public research should also be conducted on whether an electors' voting behaviour would change in instances where there is a possibility for his/her vote to move to another constituency. At present, the public is aware of the possibility of wasted votes under FPTP and this is reflected in national media discussions at election time. Whether, and to what extent, voting behaviour might alter in a system without the significant numbers of wasted votes created by FPTP is unknown.

5.3 Concluding Remarks

This project has presented evidence to support the hypothesis that Concentrated Vote would increase proportionality in UK general elections. The differences in terms of proportionality between FPTP and Concentrated Vote are significant in all respects measured. The web interface of visualisations developed for this project presents these differences in an easily understood format, which may assist in any public discussion of proportionality and electoral reform. While the future for Concentrated Vote as a possible electoral system for the UK is, as with all things requiring political momentum, unclear, the strength of Concentrated Vote as a tool to achieve proportional election results is clear.

Appendix A

Derivation of Concentrated Vote Constraints

A.1 Constituency Constraint

$$\sum_i \sum_j \frac{\partial L}{\partial X_{ij}} \cdot \frac{\partial g_l}{\partial X_{ij}} = 0 \tag{A.1}$$

$$\text{Gradient } \frac{\partial g_l}{\partial X_{ij}} = \delta_{il} R_{lj}, \text{ where } \delta_{il} \text{is the Kronecker delta} \tag{A.2}$$

$$\sum_i \sum_j R_{ij}(X_{ij} - \lambda_i - \mu_j) R_{ij} \delta_{il} = 0 \tag{A.3}$$

$$\sum_j R_{lj}(X_{lj} - \lambda_l - \mu_j) = 0 \tag{A.4}$$

$$\sum_j R_{lj} X_{lj} - \lambda_l \sum_j R_{lj} - \sum_j \mu_j R_{lj} = 0 \tag{A.5}$$

Substitute in equation 2.5

$$c_l - \lambda_l \sum_j R_{lj} - \sum_j \mu_j R_{lj} = 0 \tag{A.6}$$

A.2 Party Constraint

$$\sum_j \sum_i \frac{\partial L}{\partial X_{ij}} \cdot \frac{\partial h_m}{\partial X_{ij}} = 0 \tag{A.7}$$

$$\text{Gradient } \frac{\partial h_m}{\partial X_{ij}} = \delta_{jm} R_{im}, \text{ where } \delta_{il} \text{is the Kronecker delta} \tag{A.8}$$

$$\sum_j \sum_i R_{ij}(X_{ij} - \lambda_i - \mu_j) R_{im} \delta_{jm} = 0 \tag{A.9}$$

$$\sum_i R_{im}(X_{im} - \lambda_i - \mu_m) = 0 \tag{A.10}$$

$$\sum_i R_{im} X_{im} - \mu_m \sum_i R_{im} - \sum_i \lambda_i R_{im} = 0 \tag{A.11}$$

Substitute in equation 2.6

$$p_m - \sum_i \lambda_i R_{im} - \mu_m \sum_i R_{im} = 0 \tag{A.12}$$

Appendix B

Screenshots of Web Interface

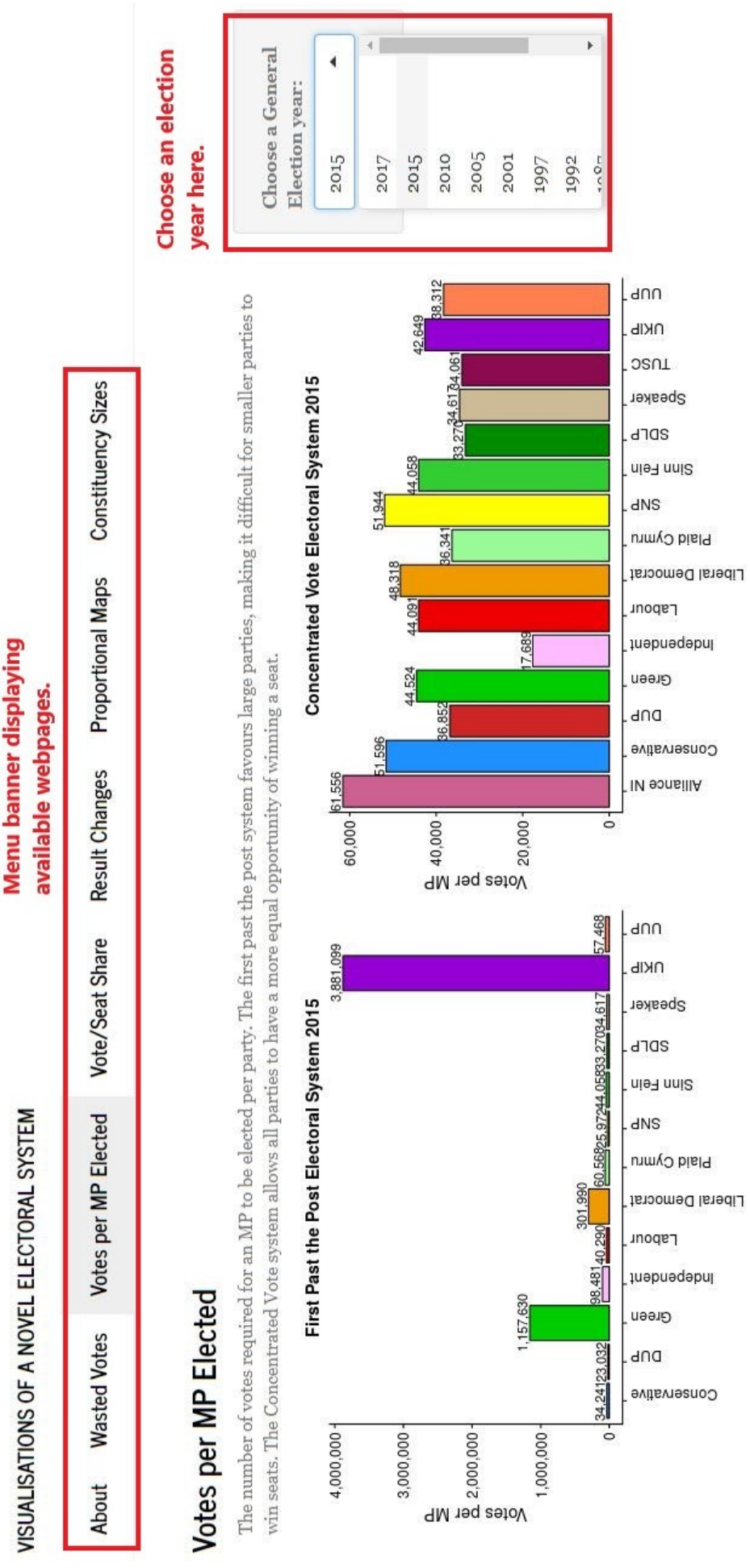


Figure B.1: Annotated Screenshot of Web Interface Highlighting Functionality

VISUALISATIONS OF A NOVEL ELECTORAL SYSTEM | About | Wasted Votes | Votes per MP Elected | Vote/Seat Share | Result Changes | Proportional Maps | Constituency Sizes

What is the "Concentrated Vote" Electoral System?

Overview | Further Details

Background

Majoritarian electoral systems are the oldest and simplest systems in use in democratic states and, despite a number of notable drawbacks, the most popular. They can be described as "winner takes all" and produce outcomes that reflect the views of the majority of a population, even where the majority is a small one. Majority systems are often criticised for producing results that disproportionately favour certain political parties and results that are unrepresentative of the whole of a country's voting population. For example, in the UK, which uses the majoritarian first past the post (FPTP) electoral method, the results of the 2015 general election were widely described as the most disproportionate in history. In the UK context, these criticisms are notable as they come only four years after the UK voted to reject the Alternative Vote, a proportional system, which would have replaced FPTP.

The Concentrated Vote Electoral System

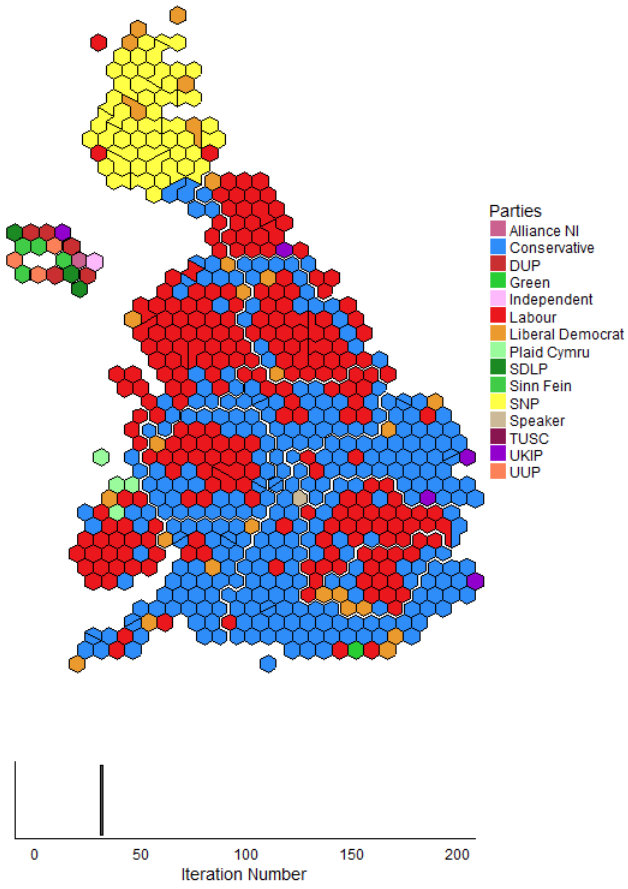
The Concentrated Vote is an unpublished voting system developed by Dr. David Sterratt at the University of Edinburgh. Concentrated Vote is an alternative voting system that has been applied to the UK general election data from 1979-2017 as part of an MSc project. The aim of the Concentrated Vote electoral system is to address the issue of disproportionality with the FPTP system, currently used in the UK, while maintaining its simplicity for voters. Votes are cast in the same way as the FPTP system, with voters required to select a single candidate in a constituency. Votes are then counted to give an array of votes per party in each constituency but no candidates are elected until all votes in the country or region have been counted.

The system works by first eliminating the candidate with the smallest number of votes in a given constituency and then shifting these votes to a constituency where the eliminated candidate's party has more votes. The process works by eliminating candidates one-by-one, which leads to a concentration of votes where a party started out strong.

The Graphic

The graphic on the right demonstrates this process of seats changing as votes are shifted between constituencies When all but a single candidate has been eliminated in a constituency, the candidate is then elected. At every stage of this process, the number of votes per constituency and per party remains unchanged. The process ends when there is no candidate that can be eliminated without breaking one of the constraints relating to the number of votes per constituency and per party.

Concentrated Vote Process Transfering Votes Between Constituencies



Choose a General Election year:

2015

Figure B.2: Screenshot of Overview Tab of Homepage

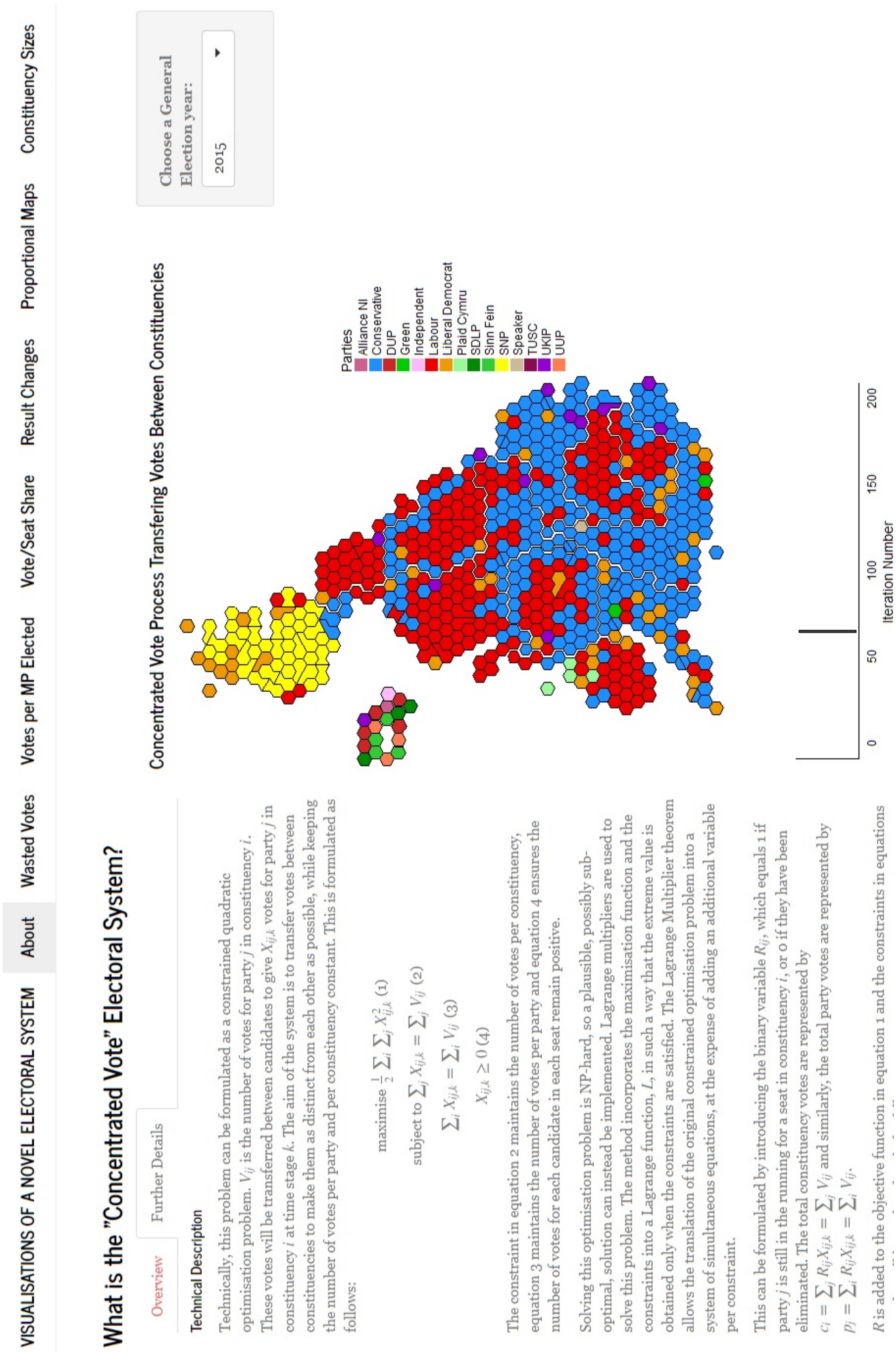
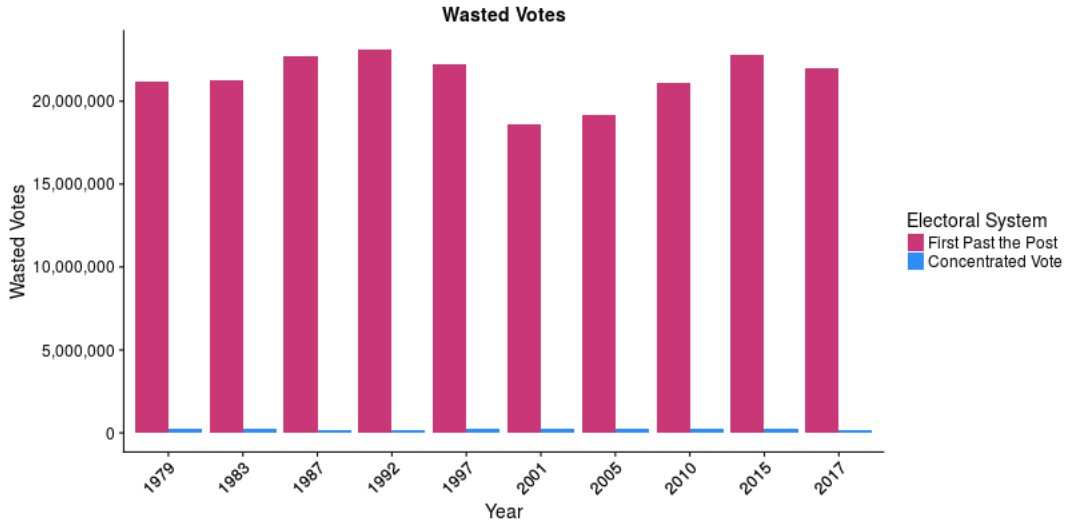


Figure B.3: Screenshot of Further Details Tab of Homepage

VISUALISATIONS OF A NOVEL ELECTORAL SYSTEM | About | Wasted Votes | Votes per MP Elected | Vote/Seat Share | Result Changes | Proportional Maps | Constituency Sizes

Wasted Votes

In the first past the post system, wasted votes are votes cast for losing candidates or votes cast for winning candidates in excess of the number required for victory. In the Concentrated Vote system, wasted votes are those left over when the process of elimination cannot continue without breaking one of the constraints relating to the number of votes per constituency and per party.



Details of Wasted Votes

Year	Votes Wasted		% Wasted	
	FPTP	CV	FPTP	CV
2017	22,016,215	165,516	68.42%	0.51%
2015	22,813,937	234,334	74.33%	0.76%
2010	21,122,519	287,814	71.15%	0.97%
2005	19,150,430	263,684	70.75%	0.97%
2001	18,573,027	212,848	70.45%	0.81%
1997	22,219,761	221,747	71.03%	0.71%
1992	23,099,145	189,059	68.72%	0.56%
1987	22,690,160	150,629	69.75%	0.46%
1983	21,297,953	282,310	69.63%	0.92%
1979	21,222,055	254,513	67.96%	0.82%

Showing 1 to 10 of 10 entries

Figure B.4: Screenshot of Wasted Votes Webpage

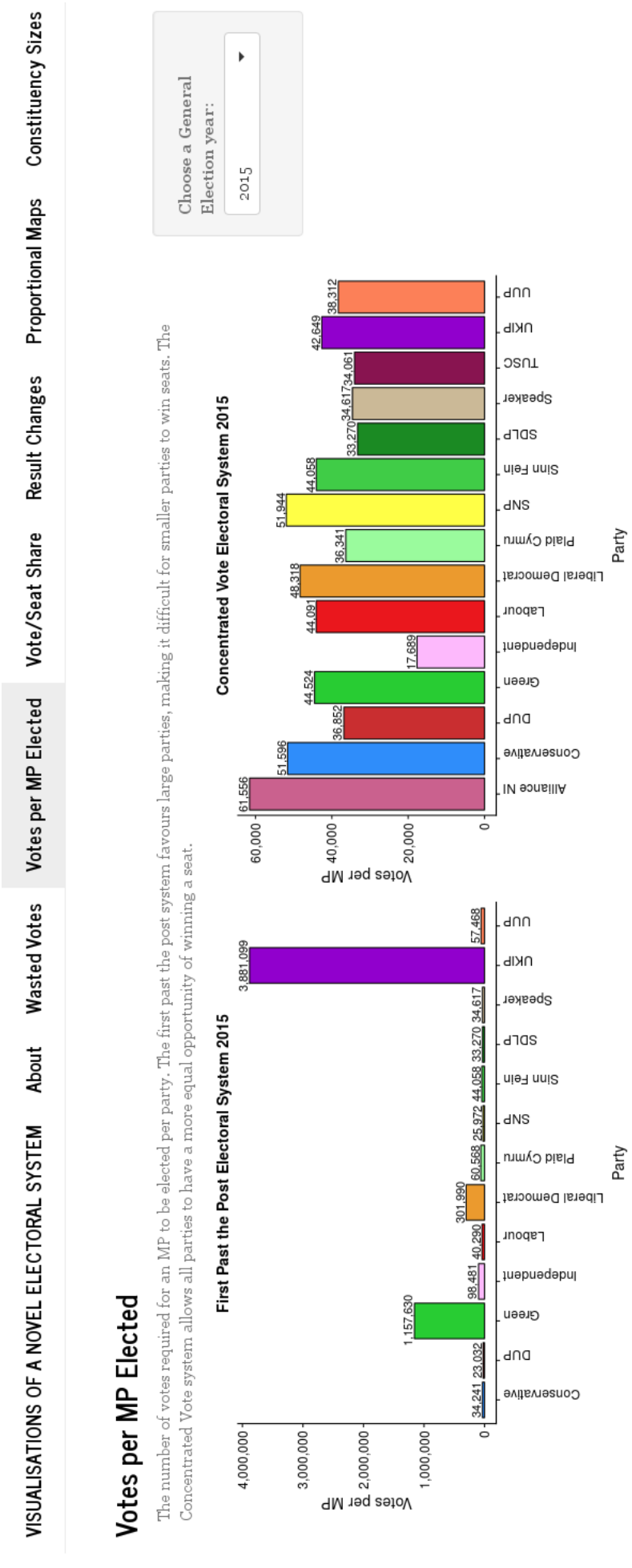


Figure B.5: Screenshot of Votes per MP Elected Webpage

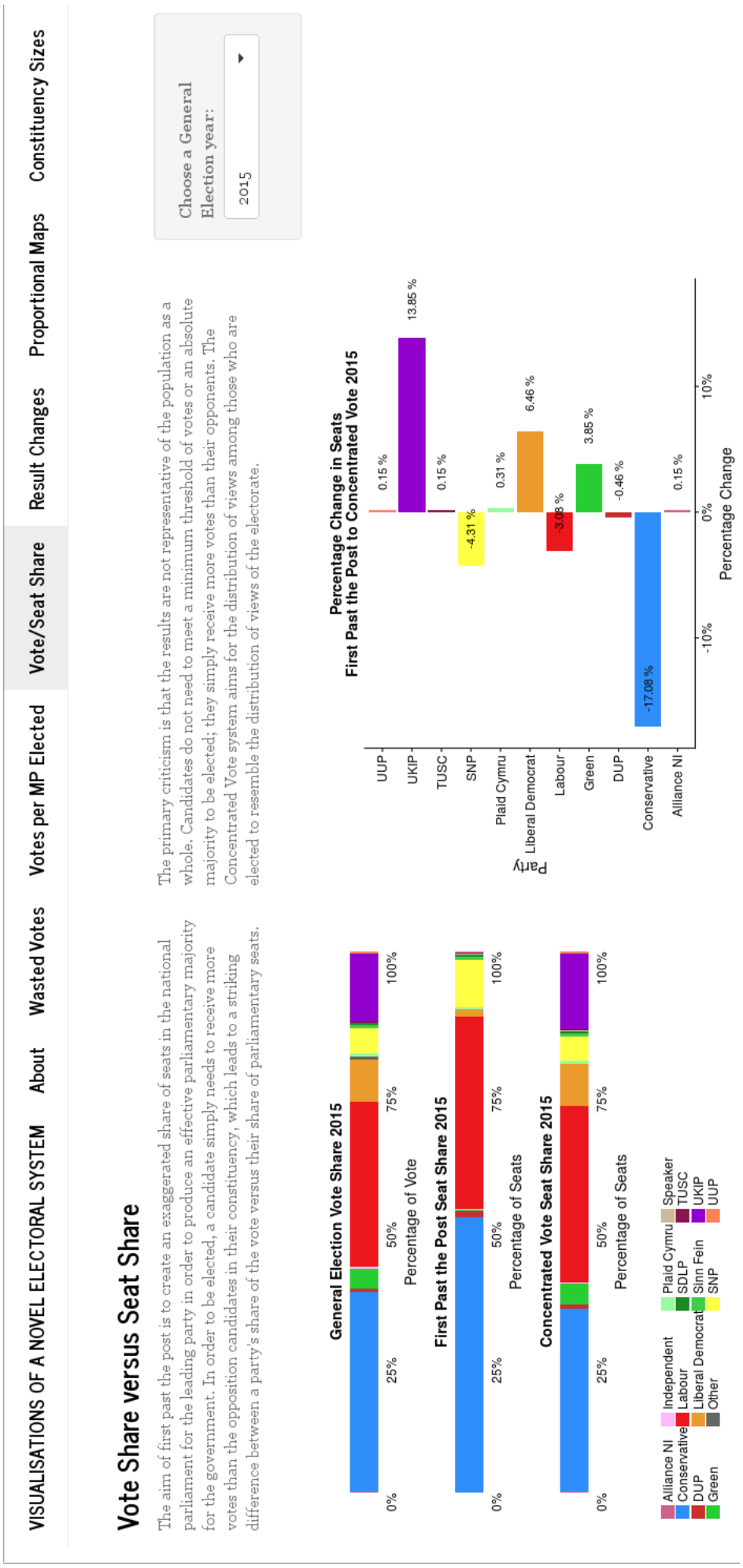


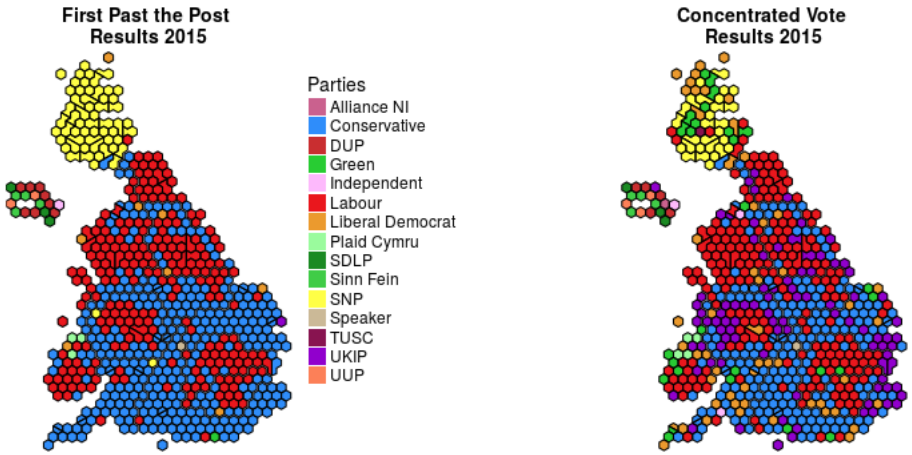
Figure B.6: Screenshot of Vote/Seat Share Webpage

Figure B.7: Screenshot of Result Changes Webpage

VISUALISATIONS OF A NOVEL ELECTORAL SYSTEM | About | Wasted Votes | Votes per MP Elected | Vote/Seat Share | Result Changes | Proportional Maps | Constituency Sizes

Cartograms of UK General Election Results

A cartogram is a map in which geographic region sizes appear in proportion to their population and are particularly relevant for election data as on an ordinary geographic map constituency size is not proportional to its population.



Changes to Party Seats

Show 10 entries Search:

Party	% of Vote	FPTP % of Seats	CV% of Seats	+/- Seats
Alliance	0.20%	0%	0.20%	1
Conservative	36.82%	50.80%	33.70%	-111
DUP	0.6%	1.20%	0.80%	-3
Green	3.77%	0.20%	4%	25
Independent	0.06%	0.20%	0.20%	0
Labour	30.46%	35.70%	32.60%	-20
Liberal Democrat	7.87%	1.20%	7.70%	42
Plaid Cymru	0.59%	0.50%	0.80%	2
SDLP	0.33%	0.50%	0.50%	0
Sinn Fein	0.57%	0.60%	0.60%	0

Showing 1 to 10 of 15 entries Previous 1 2 Next

Choose a General Election year: 2015

Figure B.8: Screenshot of Proportional Maps Webpage

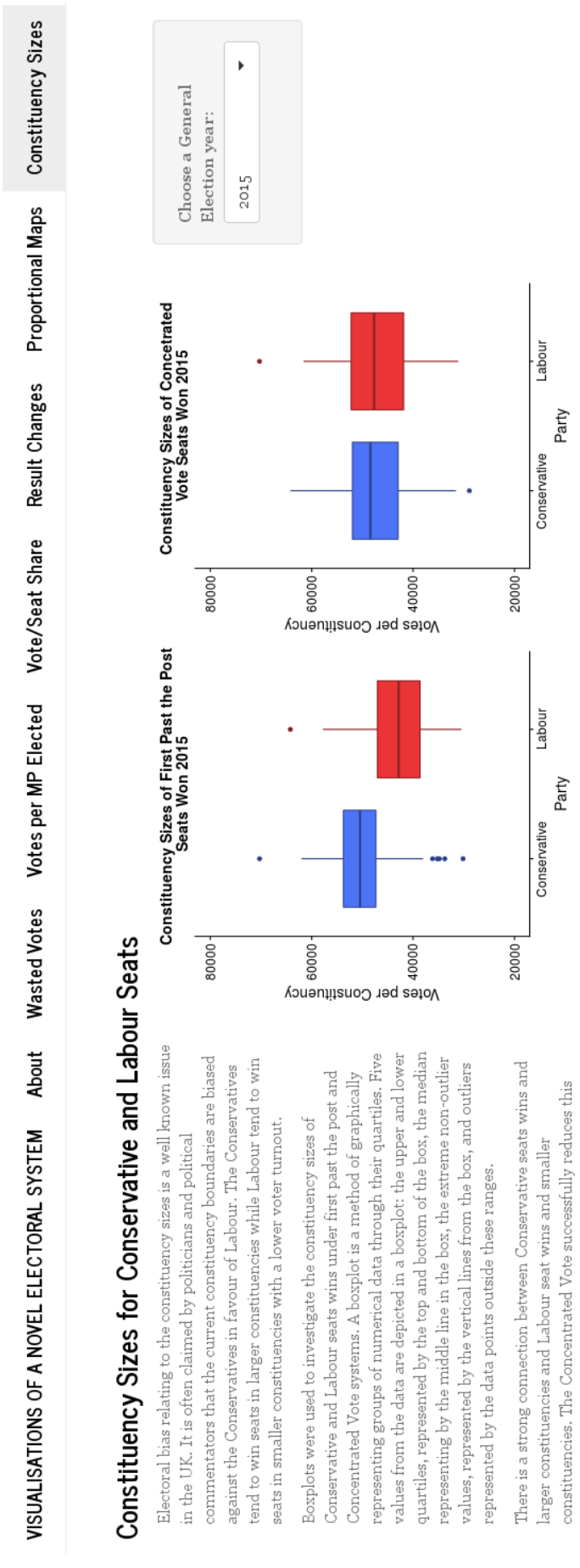


Figure B.9: Screenshot of Constituency Sizes Webpage

Appendix C

Questionnaire for Interface Usability

Questionnaire for Interface Usability

Please rate your reaction to the user interface (https://maca.shinyapps.io/mscproject).

* Required

1. **Overall reaction to the interface.** *
   Mark only one oval.

	1	2	3	4	5	6	7	8	9	
Terrible	◯	◯	◯	◯	◯	◯	◯	◯	◯	Wonderful

2. **Overall reaction to the interface.** *
   Mark only one oval.

	1	2	3	4	5	6	7	8	9	
Frustrating	◯	◯	◯	◯	◯	◯	◯	◯	◯	Satisfying

3. **Overall reaction to the interface.** *
   Mark only one oval.

	1	2	3	4	5	6	7	8	9	
Dull	◯	◯	◯	◯	◯	◯	◯	◯	◯	Stimulating

4. **Overall reaction to the interface.** *
   Mark only one oval.

	1	2	3	4	5	6	7	8	9	
Difficult	◯	◯	◯	◯	◯	◯	◯	◯	◯	Easy

5. **Overall reaction to the interface.** *
   Mark only one oval.

	1	2	3	4	5	6	7	8	9	
Inadequate power	◯	◯	◯	◯	◯	◯	◯	◯	◯	Adequate power

6. **Overall reaction to the interface.** *
   Mark only one oval.

	1	2	3	4	5	6	7	8	9	
Rigid	◯	◯	◯	◯	◯	◯	◯	◯	◯	Flexible

7. **Other thoughts or comments.**

Screen Design

Please rate your reaction to the screen design.

8. **Screen layouts were helpful.** *
   Mark only one oval.

	1	2	3	4	5	6	7	8	9	
Never	◯	◯	◯	◯	◯	◯	◯	◯	◯	Always

9. **The amount of information that can be displayed on screen.** *
   Mark only one oval.

	1	2	3	4	5	6	7	8	9	
Inadequate	◯	◯	◯	◯	◯	◯	◯	◯	◯	Adequate

10. **Organisation of information.** *
    Mark only one oval.

	1	2	3	4	5	6	7	8	9	
Confusing	◯	◯	◯	◯	◯	◯	◯	◯	◯	Clear

11. **Sequence of screens** *
    Mark only one oval.

	1	2	3	4	5	6	7	8	9	
Confusing	◯	◯	◯	◯	◯	◯	◯	◯	◯	Clear

12. **Other thoughts or comments on screen design.**

Learning

Please rate your reaction to learning the interface.

13. **Learning to operate the system.** *
Mark only one oval.

	1	2	3	4	5	6	7	8	9	
Difficult	◯	◯	◯	◯	◯	◯	◯	◯	◯	Easy

14. **Exploration of features by trial and error.** *
Mark only one oval.

	1	2	3	4	5	6	7	8	9	
Discouraging	◯	◯	◯	◯	◯	◯	◯	◯	◯	Encouraging

15. **Performing tasks is straightforward.** *
Mark only one oval.

	1	2	3	4	5	6	7	8	9	
Never	◯	◯	◯	◯	◯	◯	◯	◯	◯	Always

16. **Use of terms throughout system.** *
Mark only one oval.

	1	2	3	4	5	6	7	8	9	
Inconsistent	◯	◯	◯	◯	◯	◯	◯	◯	◯	Consistent

17. **Other thoughts or comments on learning.**

System Capabilities

Please rate your reaction to the system capabilities.

18. **System speed.** *
Mark only one oval.

	1	2	3	4	5	6	7	8	9	
Too slow	◯	◯	◯	◯	◯	◯	◯	◯	◯	Fast enough

19. **Response time for most operations.**
Mark only one oval.

	1	2	3	4	5	6	7	8	9	
Too slow	◯	◯	◯	◯	◯	◯	◯	◯	◯	Fast enough

20. **System designed for all levels of users.** *
Mark only one oval.

	1	2	3	4	5	6	7	8	9	
Never	◯	◯	◯	◯	◯	◯	◯	◯	◯	Always

21. **Other thoughts or comments on system capabilities.**

_____

_____

_____

_____

_____

Appendix D

Questionnaire Results

Questionnaire for Interface Usability

QUESTIONS RESPONSES 12

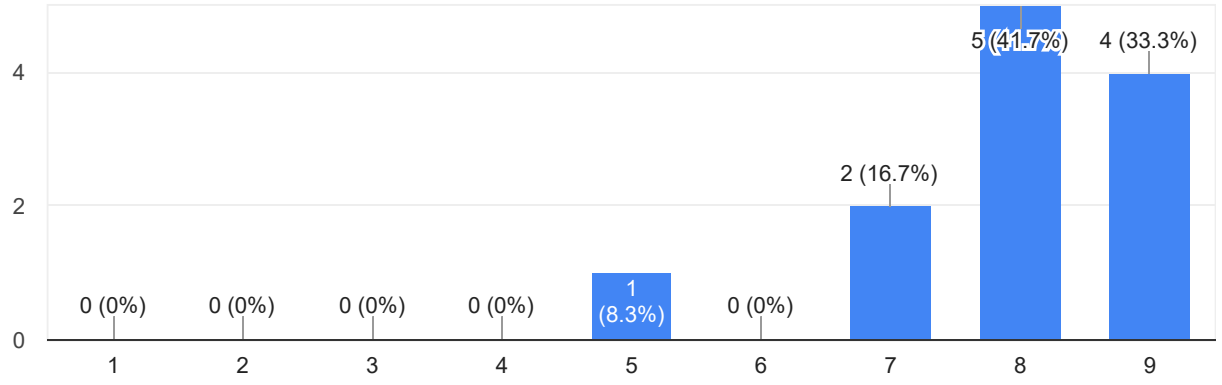
12 responses

SUMMARY INDIVIDUAL

Accepting responses

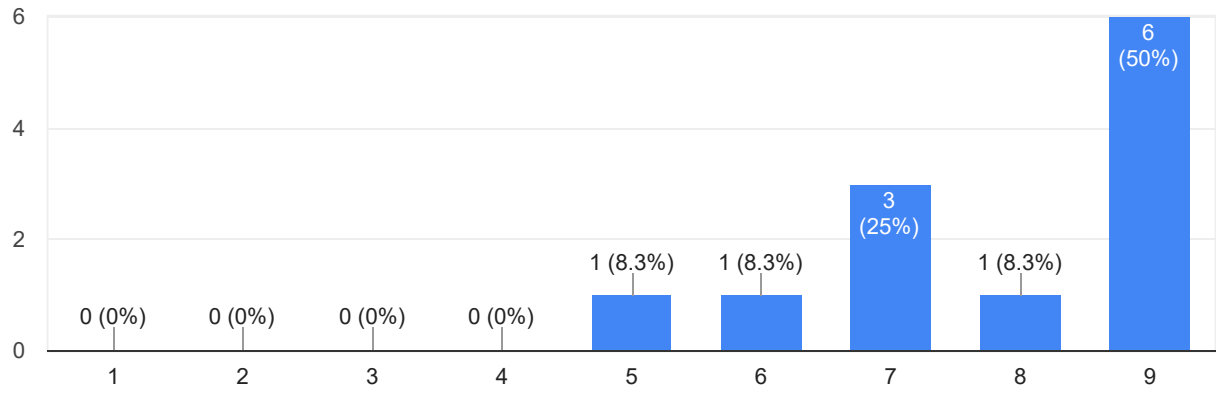
Overall reaction to the interface.

12 responses



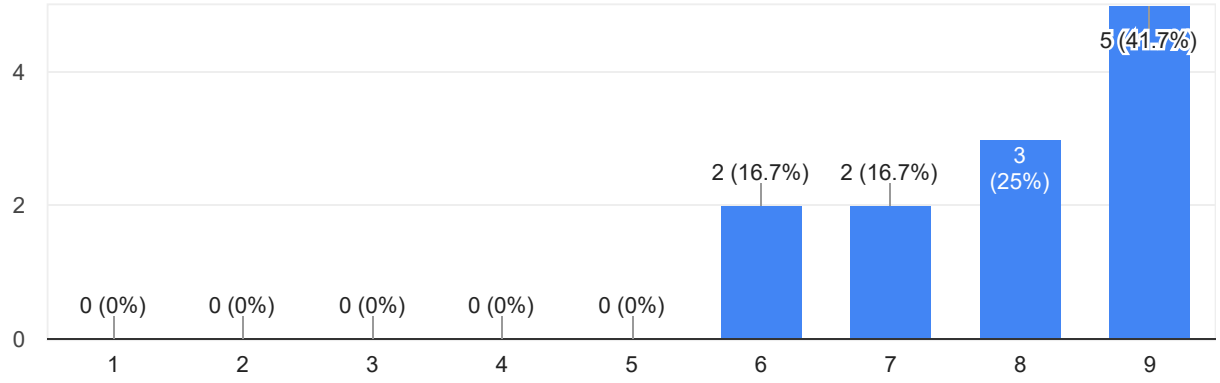
Overall reaction to the interface.

12 responses



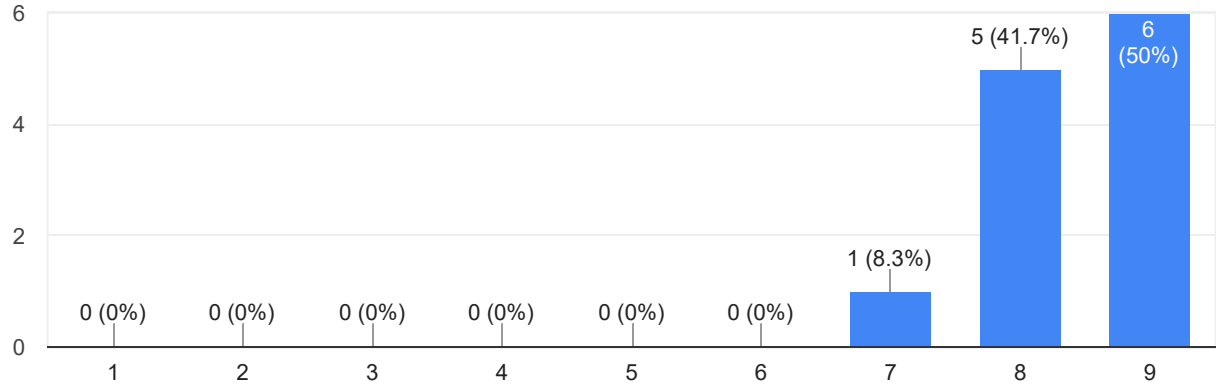
Overall reaction to the interface.

12 responses



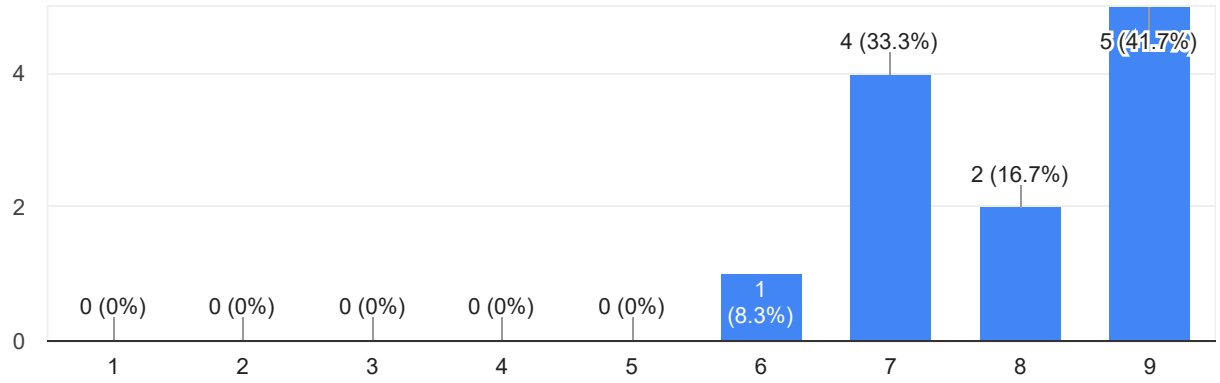
Overall reaction to the interface.

12 responses



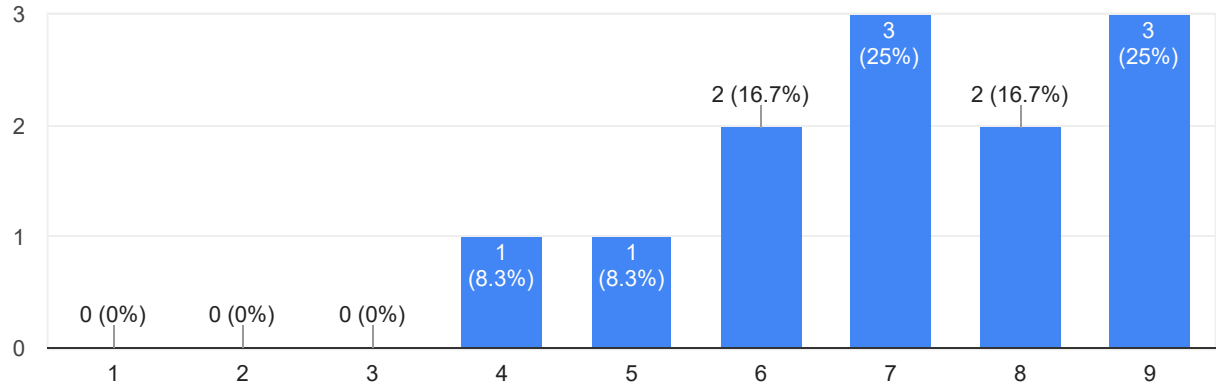
Overall reaction to the interface.

12 responses



Overall reaction to the interface.

12 responses



Other thoughts or comments.

8 responses

a lot of information being conveyed very interesting

Very visually appealing and easy to navigate

excellent layout, easy to navigate through each tab and easy to see each graph. good use of highlighting tabs to know which one you are choosing. spacing of text and graphs good.

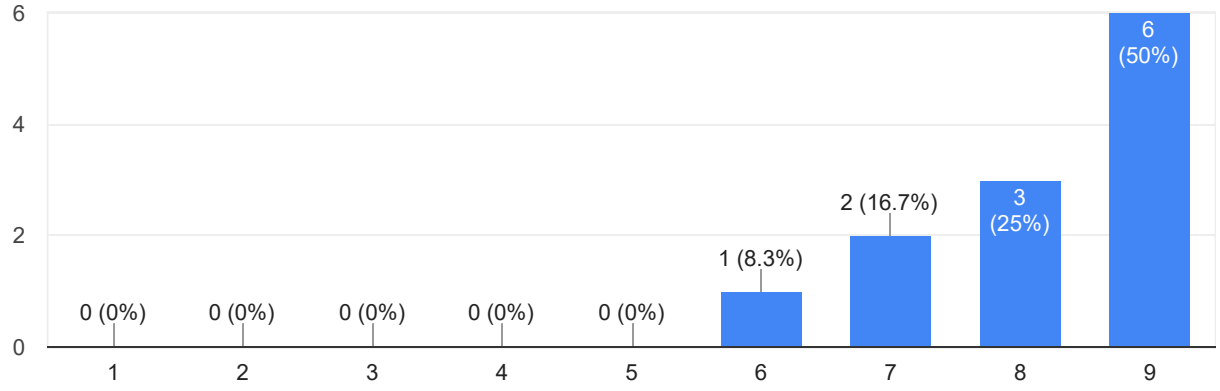
Text could be broken up a bit more, maybe with bullet points.

On the home page the colourful map grabbed my attention, but to know what the map is doing I had to read the big block of text. I think a title for the graphic like like "how the 'Concentrated Vote' system would change election results" would be easier to get.

Screen Design

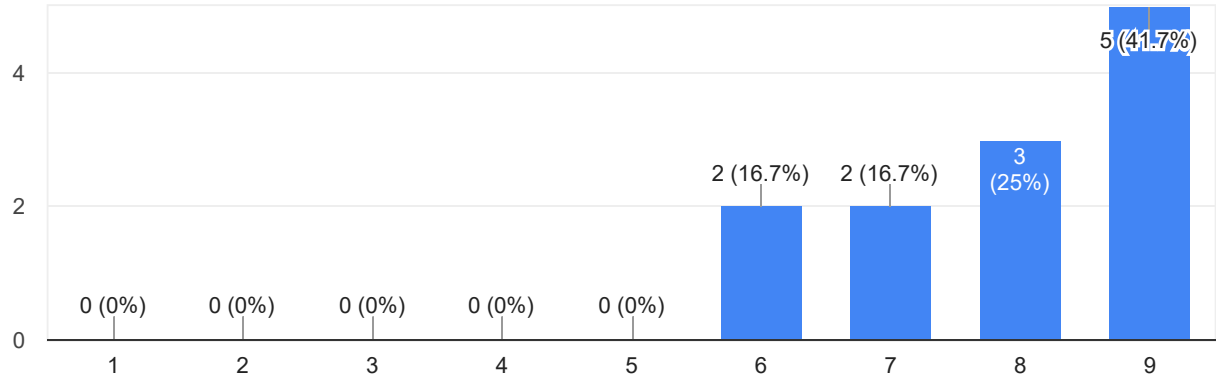
Screen layouts were helpful.

12 responses



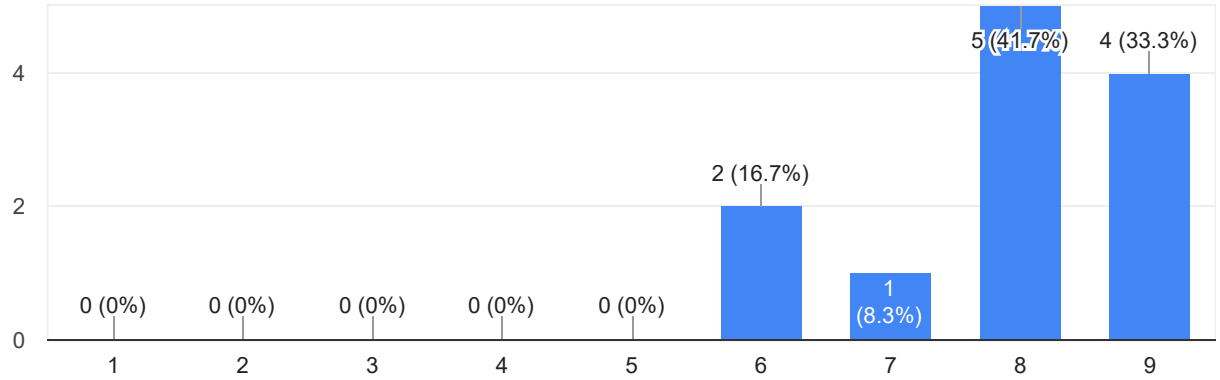
The amount of information that can be displayed on screen.

12 responses



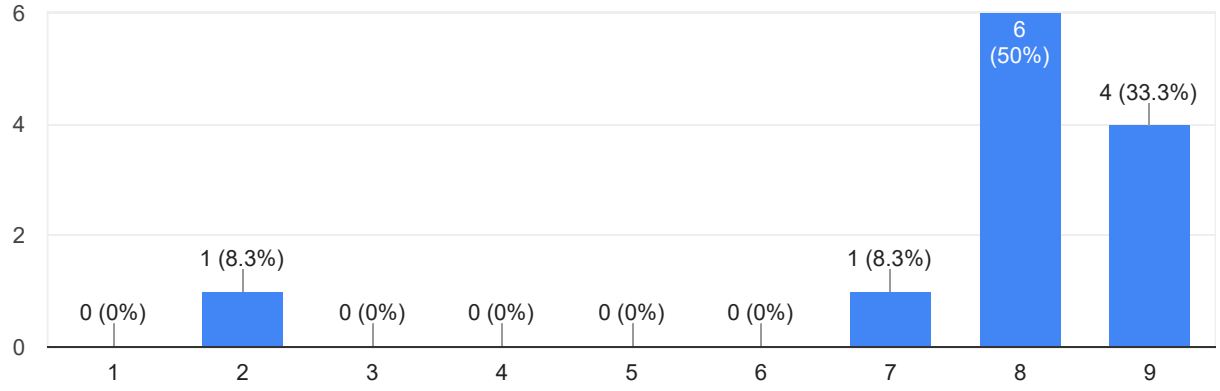
Organisation of information.

12 responses



Sequence of screens

12 responses



Other thoughts or comments on screen design.

4 responses

controls were consistent colours were good titles followed a pattern

Some of the graphs could be more spread out to more easily differentiate between them (eg seat vs vote page)

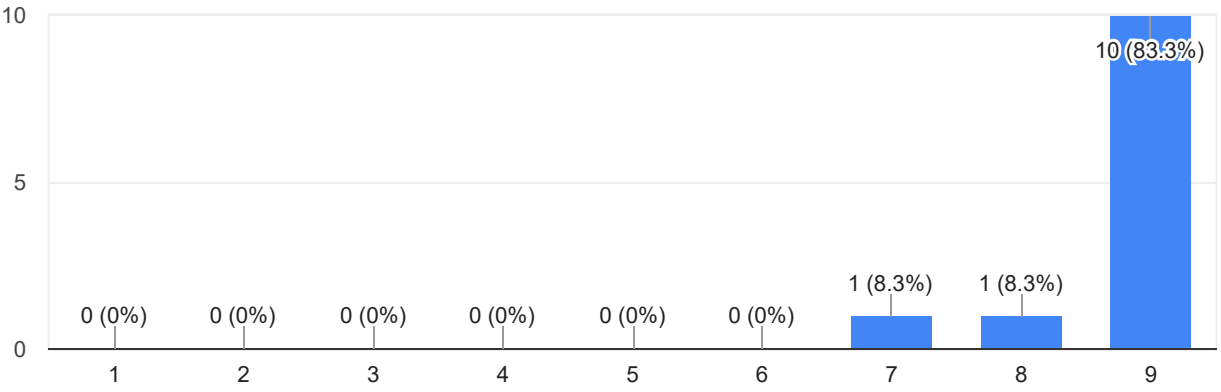
Was there an intended sequence to the screens? I wasn't sure. You could add some little arrows to the top menu to indicate an order if needed eg:
About --> Wasted Votes --> etc.
You could also intro page to be the home page that gives a really brief intro to the project, maybe has the map to grab attention, and says click through the tabs in order to see how different elements or the voting process are affected. And maybe mention that data is available for multiple election years with the drop down, which I didn't notice at first.

Structured layout, user friendly.

Learning

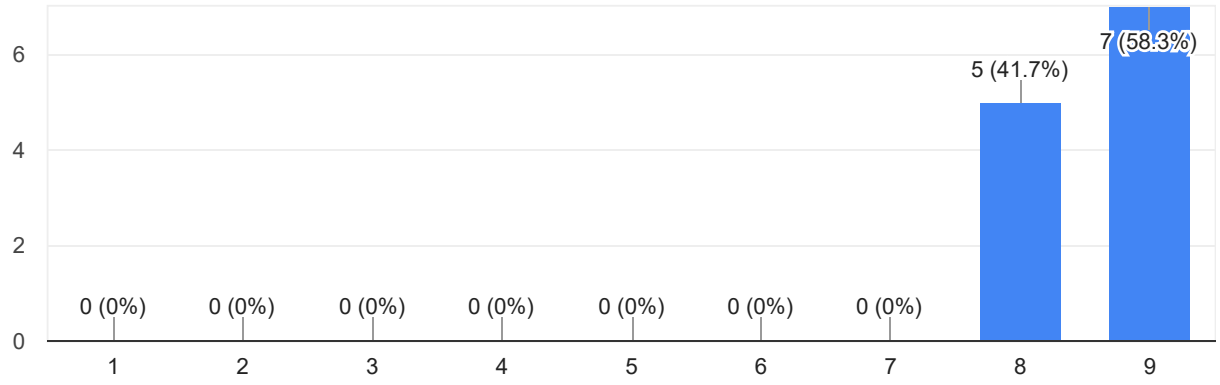
Learning to operate the system.

12 responses



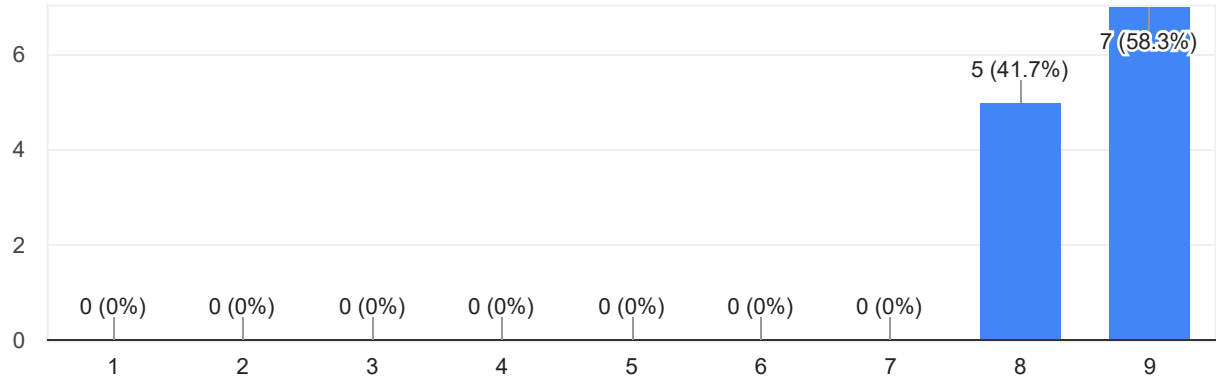
Exploration of features by trial and error.

12 responses



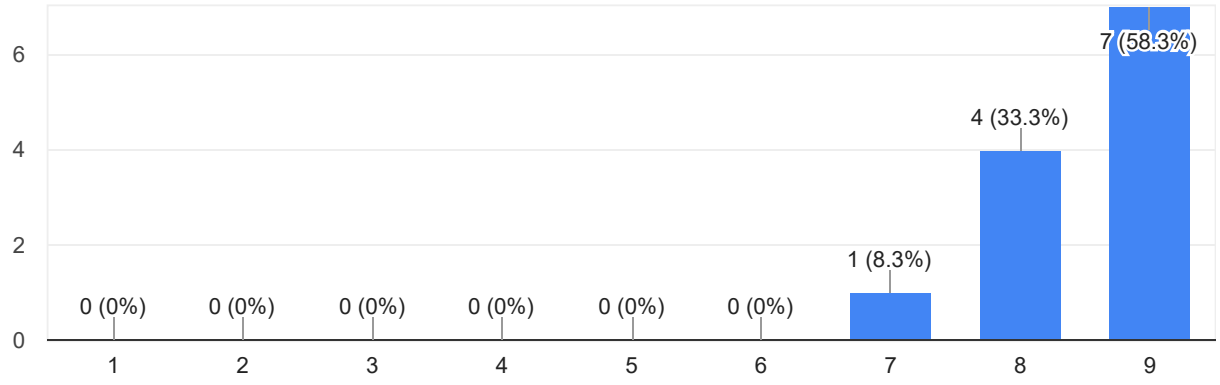
Performing tasks is straightforward.

12 responses



Use of terms throughout system.

12 responses



Other thoughts or comments on learning.

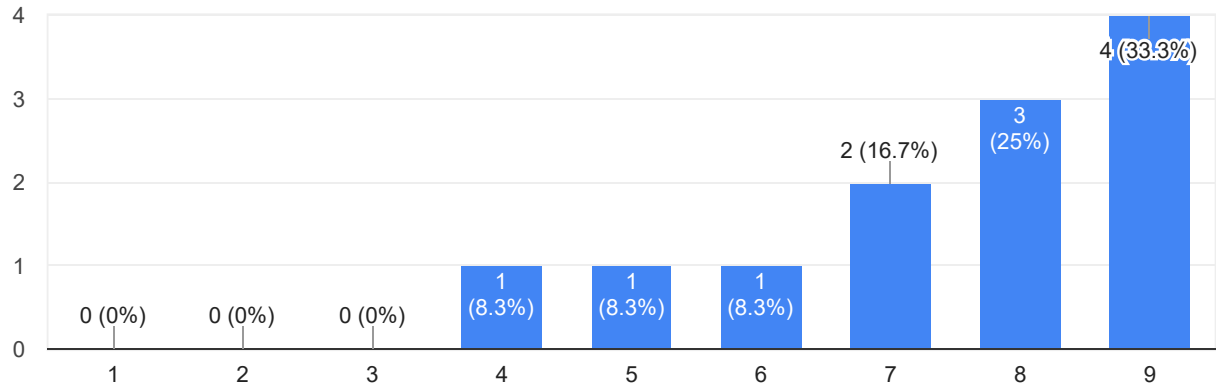
0 responses

No responses yet for this question.

System Capabilities

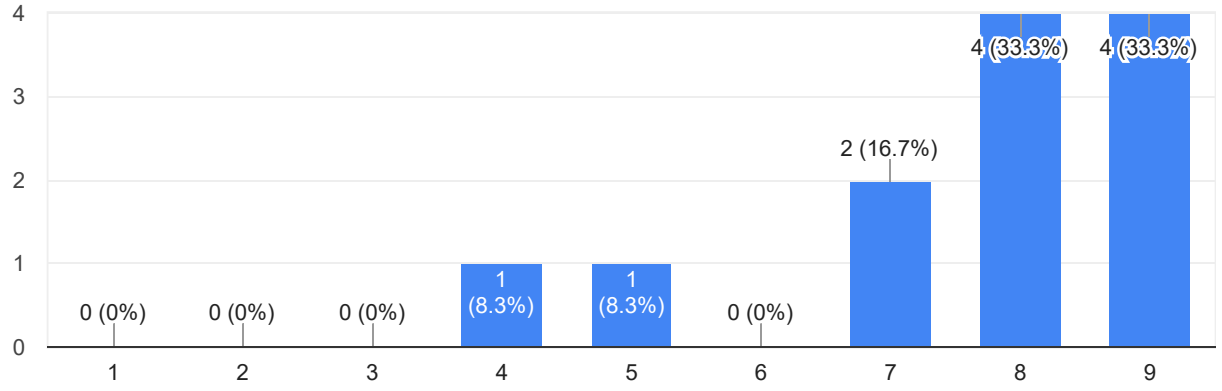
System speed.

12 responses



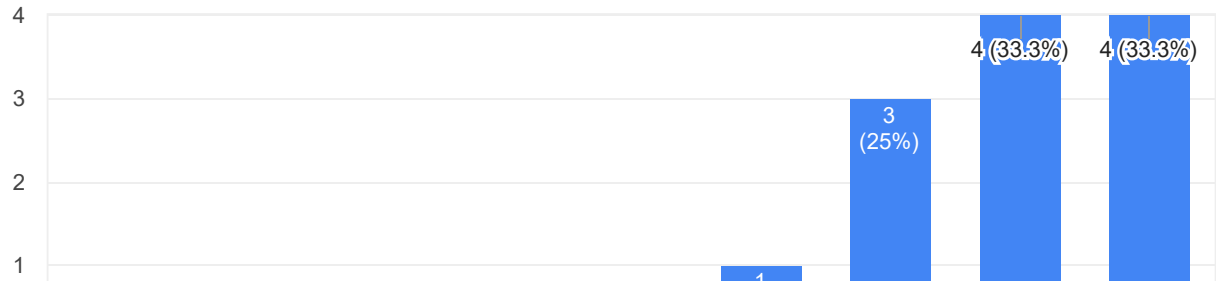
Response time for most operations.

12 responses



System designed for all levels of users.

12 responses



Other thoughts or comments on system capabilities.

2 responses

Great work!

Appendix E

Votes per MP Elected

Votes per MP Elected, General Election 2017		
Party	First Past the Post	Concentrated Vote
Alliance NI	-	64,553
Conservative	42,818	52,942
DUP	29,232	41,759
Green Party	523,693	43,641
Independent	175,689	16,148
Labour	49,430	47,423
Liberal Democrat	196,761	49,190
Other	-	29,673
Plaid Cymru	40,694	32,555
SDLP	-	47,709
Sinn Fein	34,131	47,783
SNP	27,931	46,551
Speaker	34,299	34,299
UKIP	-	42,097
UUP	-	41,640

Table E.1: Votes per MP Elected for the 2017 General Election

Votes per MP Elected, General Election 2010		
Party	First Past the Post	Concentrated Vote
Alliance NI	42,762	42,762
British National	-	37,621
Christian Party	-	18,622
Conservative	34,979	49,326
DUP	21,027	33,643
English Democrat	-	32,413
Green Party	285,612	40,802
Independent	175,604	21,181
Labour	33,359	42,607
Liberal Democrat	119,934	46,824
Plaid Cymru	55,131	33,079
Respect	-	33,251
SDLP	36,990	36,990
Sinn Fein	34,388	34,388
SNP	81,898	40,949
Speaker	22,860	22,860
TUV	-	26,300
UCUNF	-	51,181
UKIP	-	39,977

Table E.2: Votes per MP Elected for the 2010 General Election

Votes per MP Elected, General Election 2005		
Party	First Past the Post	Concentrated Vote
Alliance NI	-	28,291
British National	-	32,124
Conservative	44,508	46,888
DUP	26,873	40,309
Green Party	285,612	31,576
Independent	118,560	20,505
Health Concern	18,739	-
Labour	26,821	38,392
Liberal Democrat	94,621	43,832
Liberal	-	19,068
Plaid Cymru	58,279	29,140
Respect	68,094	34,047
SDLP	41,875	41,875
Sinn Fein	34,906	43,633
SNP	68,711	37,479
Speaker	15,153	15,153
Scottish Socialist	-	43,514
UKIP	-	33,545
UUP	126,323	31,581
Veritas	-	40,607

Table E.3: Votes per MP Elected for the 2005 General Election

Votes per MP Elected, General Election 2001		
Party	First Past the Post	Concentrated Vote
Alliance NI	-	28,999
British National	-	23,598
Conservative	50,338	44,447
DUP	36,400	36,400
Green Party	285,612	33,295
Health Concern	28,487	28,487
Labour	26,039	36,866
Liberal Democrat	92,527	42,579
Plaid Cymru	48,973	32,649
SDLP	56,622	56,622
Sinn Fein	43,983	43,983
SNP	92,859	30,953
Speaker	16,053	16,053
Scottish Socialist	-	24,169
Socialist Alliance	-	29,978
Socialist Labour	-	28,644
Ulster Unionist	36,140	43,368
UKIP	-	32,515

Table E.4: Votes per MP Elected for the 2001 General Election

Votes per MP Elected, General Election 1997		
Party	First Past the Post	Concentrated Vote
Alliance NI	-	31,486
British National	-	35,832
Conservative	58,188	51,897
DUP	53,674	35,783
Green Party	-	31,996
Independent	120,552	29,354
Labour	32,340	45,211
Liberal	-	45,166
Liberal Democrat	113,977	50,902
Natural Law	-	30,604
People's Labour	-	19,332
Plaid Cymru	40,258	40,258
Referendum	-	35,298
SDLP	63,605	47,704
Sinn Fein	63,461	42,307
SNP	103,592	41,437
Speaker	23,969	23,969
Socialist Labour	-	52,109
Ulster Unionist	25,835	43,058
UKIP	-	35,241
UK Unionist	12,817	-

Table E.5: Votes per MP Elected for the 1997 General Election

Votes per MP Elected, General Election 1992		
Party	First Past the Post	Concentrated Vote
Alliance NI	-	34,333
Conservative	41,943	58,236
DUP	34,346	34,346
Green Party	-	34,114
Independent Labour	-	33,461
Labour	42,734	47,095
Liberal	-	32,372
Liberal Democrat	300,516	50,935
Natural Law	-	31,284
Plaid Cymru	37,508	37,508
SDLP	46,111	61,482
Sinn Fein	63,461	39,146
SNP	209,855	41,971
Speaker	22,251	22,251
Ulster Popular Unionist	19,305	-
Ulster Unionist	30,117	45,175

Table E.6: Votes per MP Elected for the 1992 General Election

Votes per MP Elected, General Election 1987		
Party	First Past the Post	Concentrated Vote
Alliance NI	-	36,336
Conservative	36,631	54,947
DUP	28,547	42,821
Green Party	-	29,918
Independent Labour	26,176	26,176
Labour	43,888	46,113
Liberal	246,734	50,536
Plaid Cymru	41,200	41,200
SDLP	51,362	51,362
Sinn Fein	83,389	83,389
SNP	138,958	41,687
Social Democrat	629,431	48,418
Speaker	24,188	24,188
The Workers NI	-	19,294
Ulster Popular Unionist	18,420	18,420
Ulster Unionist	30,692	39,461

Table E.7: Votes per MP Elected for the 1987 General Election

Votes per MP Elected, General Election 1983		
Party	First Past the Post	Concentrated Vote
Alliance NI	-	26,130
Alliance (Liberal)	235,966	46,675
Alliance (SDP)	584,212	44,939
British National	-	14,287
Conservative	32,821	50,641
DUP	50,916	38,187
Ecology	-	26,622
Independent Labour	-	16,447
Labour	40,411	44,452
National Front	-	27,053
Official Unionist	23,701	52,141
Plaid Cymru	62,253	41,502
SDLP	137,012	45,671
Sinn Fein	102,701	51,351
SNP	164,080	36,462
Speaker	22,292	22,292
Ulster Popular Unionist	22,861	22,861

Table E.8: Votes per MP Elected for the 1983 General Election

Votes per MP Elected, General Election 1979		
Party	First Past the Post	Concentrated Vote
Alliance NI	-	82,892
Conservative	40,409	55,912
DUP	23,658	70,975
Ecology	-	19,959
Independent	32,047	-
Labour	42,928	47,345
Liberal	391,909	41,452
National Front	-	24,963
Official Unionist	50,916	63,645
Plaid Cymru	66,222	33,111
Republican Clubs	-	12,100
SDLP	137,010	45,670
Sinn Fein	-	51,351
SNP	252,130	33,617
Speaker	27,035	27,035
Ulster Unionist	36,989	36,989
United Ulster Unionist	39,856	39,856
Workers Revolutionary	-	13,531

Table E.9: Votes per MP Elected for the 1979 General Election

Appendix F

Marginal Seats

FPTP Marginal Seat Win Changes in Concentrated Vote					
Constituency	First Past the Post Winner	FPTP Votes	Concentrated Vote Winner	FPTP Votes	CV Votes
Bedford	Conservative	19,625	Labour	18,528	45,957
Bolton West	Conservative	19,744	Labour	18,943	48,271
Brighton, Kemptown	Conservative	18,428	Labour	17,738	45,164
Bury North	Conservative	18,970	Labour	18,592	45,230
Cambridge	Labour	18,646	Liberal Democrat	18,047	51,587
Cardiff North	Conservative	21,709	Labour	19,572	50,742
Croydon Central	Conservative	22,753	Labour	22,588	52,884
East Dunbartonshire	SNP	22,093	Liberal Democrat	19,926	54,871
Eastbourne	Conservative	20,934	Liberal Democrat	20,201	52,768
Gower	Conservative	15,862	Labour	15,835	42,337
Lewes	Conservative	19,206	Liberal Democrat	18,123	50,540
Lincoln	Conservative	19,976	Labour	18,533	46,566
Morley and Outwood	Conservative	18,776	Labour	18,354	47,771
Plymouth, Sutton and Devonport	Conservative	18,120	Labour	17,597	47,857
Thornbury and Yate	Conservative	19,924	Liberal Democrat	18,429	48,570
Thurrock	Conservative	16,692	UKIP	15,718	49,454
Twickenham	Conservative	25,580	Liberal Democrat	23,563	61,804
Weaver Vale	Conservative	20,227	Labour	19,421	46,867
Ynys Mon	Labour	10,871	Plaid Cymru	10,642	34,778

Table F.1: Marginal Seats Changes (less than 10%), 2015 General Election

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