# Computational Conceptual Blending as a Model of Spatial Transfer in Natural Language

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2016

# Abstract

Research in cognitive linguistics has emphasised the importance of spatial concepts like MOTION and CONTAINMENT to reasoning about non-spatial domains. This is revealed in sentences like *The room went from hot to cold*, which use spatial language to describe a non-spatial situation. This report investigates whether computational conceptual blending can provide the mechanism whereby spatial knowledge is transferred into new domains, allowing a language learner to interpret a sentence like *The room went from hot to cold* by using their pre-existing spatial knowledge. It is shown that the structure of a *conceptual space* supports analogies between the spatial domain and other domains, allowing transfer to take place.

# Declaration

I declare that this thesis was composed by myself, that the work contained herein is my own except where explicitly stated otherwise in the text, and that this work has not been submitted for any other degree or professional qualification except as specified.

(*Daniel Worthing*)

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# Chapter 1

# Introduction

People use language to talk about about a wide variety of domains, including space, time, possession, physical properties, social relationships, and mental states. Many of our concepts only make sense within one particular domain. However, we also make use of a small collection of concepts which can be applied in many different domains. The most well-known example is MOTION, which applies to fields as seemingly unrelated as physical space, temperature, colour, emotion, possession, and information:

1. *John went from London to New York.* [physical space]
2. *The weather went from hot to cold.* [temperature]
3. *The sky went from blue to red.* [colour]
4. *The crowd went from angry to ecstatic.* [emotion]
5. *The inheritance went from John to Mary.* [possession]
6. *The news went from town to town.* [information]

Other examples of concepts which are applicable to a wide variety of domains include LOCATION, CONTAINMENT, PARTHOOD, CONNECTION and FORCE. It is a common observation in cognitive linguistics that concepts with this domain-general character tend to have a basis in physical space [Croft and Cruse, 2004, Langacker, 1987, Talmy, 2000, Lakoff and Johnson, 2008]. This raises the question of how and why spatial concepts are transferred into other domains during the course of language acquisition.

The aim of this report is to address these questions by developing a novel account of spatial transfer based on *conceptual blending*. Conceptual blending is a theory of analogy which originates in cognitive linguistics [Fauconnier and Turner, 2008]. It claims that mental domains are blended together by (a) discovering their common structure, and (b) combining them in such a way that this common structure is preserved. Blending is well-suited to modelling spatial transfer because it allows concepts to be imported from one domain into another.

Work on the mathematical foundations of conceptual blending has made it possible to implement blending in a computer-based system [Goguen, 2006, Guhe et al., 2011]. In this approach, domains are represented by theories in a many-sorted first-order logic and links between domains are represented by theory morphisms. This report follows the formalization of conceptual blending in Guhe et al. [2011], which is based on Heuristic-Driven Theory Projection (see Section 2.2.2).

## 1.1 Research Hypotheses

This report will investigate the following three hypotheses:

1. That computational cognitive blending implemented using Heuristic-Driven Theory Projection can discover consistent blends between physical space and other domains.
2. That this technique can model the transfer of spatial concepts from the spatial domain into other domains.
3. That analogies between the spatial domain and other domains are supported by the geometric structure of a conceptual space, as formulated by Gärdenfors (see Section 2.1.3).

# Chapter 2

# Background

## 2.1 Space in Linguistics

### 2.1.1 Image Schemas and Conceptual Metaphors

Cognitive linguistics is concerned with bridging the gap between the concepts communicated in language and basic bodily experiences arising from perception and action [Croft and Cruse, 2004]. For this reason, parallels between language and spatial cognition have been an important starting point for many cognitive linguists, including Langacker [1987], Talmy [2000], Lakoff [1990], Johnson [2013], and Fauconnier [1994].

A basic bodily experience is often referred to as an *image schema* [Johnson, 2013, Lakoff, 1990]. The key property of image schemas is that they originate in perceptual and motor experiences and are subsequently transferred to higher-level cognitive abilities such as reasoning and language understanding. Examples of image schemas include CONTAINMENT, PART-WHOLE, SUPPORT, and ABOVE.

The image schema which is relevant to motion sentences is the SOURCE-PATH-GOAL schema, which captures the idea of an object moving along a path towards a destination. The schema consists of four parts: a moving entity (the *trajector*), a source location, a goal location, and a path linking source and goal.


Figure 2.1: The SOURCE-PATH-GOAL image schema [Johnson, 2013].

The SOURCE-PATH-GOAL image schema is hypothesized to originate in very early experiences of objects moving in the environment, and of moving ones own limbs to reach a destination. The schema is directly encoded in language about space, such as verbs of movement and the prepositions *to* and *from*. The SOURCE-PATH-GOAL schema is also thought to be extended metaphorically to sentences like *The sky went from blue to red*, where it helps us understand a more abstract situation like change in state.

The theory of *conceptual metaphor* describes how image schemas are transferred to more abstract domains. A conceptual metaphor consists of a *source domain*, which is typically grounded in bodily experience, a *target domain*, which is more abstract, and a set of *correspondences*, which specify how knowledge about the source domain is transferred to the target domain. Examples of conceptual metaphors include GOOD IS UP, LOVE IS A FORCE and UNDERSTANDING IS SEEING.

The transfer of the SOURCE-PATH-GOAL schema from the domain of physical space to the abstract domain of change in state is supported by the conceptual metaphor CHANGE IS MOTION. This metaphor is required to understand sentences like *She's in trouble*, *The milk has gone green*, and *I've finally come out of my bad mood*. The source domain is physical space, and the target domain is physical or mental states:

Table 2.1: The STATES ARE LOCATIONS conceptual metaphor

| *Source Domain* | | *Target Domain* |
|---|---|---|
| SPACE | | STATES |
| Locations | → | States |
| Being at a location | → | Being in a state |
| The trajector | → | The object which changes state |
| The source location | → | The initial state |
| The goal location | → | The final state |
| The path linking source and goal | → | Sequence of intermediate states |

Correspondences between domains include not only entities and relations but also rules of inference [Lakoff and Núñez, 2000]. For example, the spatial domain contains the rule that 'if you have travelled from A to B and from B to C, then you have travelled from A to C'. This rule is transferred into the domain of states by the conceptual metaphor, resulting in the rule that 'if you have changed state from A to B and from B to C, then you have changed state from A to C'.

### 2.1.2 Jackendoff's Conceptual Structure

One problem with the conceptual metaphor analysis of spatial transfer is that sentences like *The sky went from blue to red* are not normally perceived to be metaphors by native speakers (compare: *"Trees are poems the earth writes upon the sky"*). Jackendoff and Aaron [1991] argue that true metaphors can be explicitly acknowledged, as in sentence (1) below. Applying the same technique to (2) leads to a non-sequitur, suggesting that it is not understood metaphorically.

1. *Of course, trees aren't poems - but if they were, you might say the earth writes them upon the sky.*
2. *?Of course, change is not motion - but if it was, you might say the sky went from blue to red.*

To explain the generality of spatial language without appealing to metaphor, Jackendoff proposes what he calls the *Thematic Relations Hypothesis*. This is the claim that SOURCE-PATH-GOAL, and other ubiquitous concepts like FORCE and PART-WHOLE, are not transferred from space into other domains, but are abstract, domain-general concepts with multiple parallel instantiations [Jackendoff, 1983, 1992].

In Jackendoff's approach to semantics, the meaning of a sentence is encoded in a symbolic language called Conceptual Structure (CS). The purpose of CS is to capture semantic generalizations, such as the similar semantics of motion sentences like *John went from London to New York*, *The sky went from blue to red* and *The money went from John to Mary*. Each of these three sentences is assigned the same abstract conceptual structure:

3. (a)
$$\left[ \mathrm{GO}_{spatial} \left[ \begin{array}{ll} \mathrm{OBJECT} & \textit{John} \\ \mathrm{PATH} & \left[ \begin{array}{ll} \mathrm{FROM} & \textit{London} \\ \mathrm{TO} & \textit{New York} \end{array} \right] \end{array} \right] \right]$$

(b)
$$\left[ \mathrm{GO}_{ident} \left[ \begin{array}{ll} \mathrm{OBJECT} & \textit{sky} \\ \mathrm{PATH} & \left[ \begin{array}{ll} \mathrm{FROM} & \textit{blue} \\ \mathrm{TO} & \textit{red} \end{array} \right] \end{array} \right] \right]$$

(c)
$$\left[ \mathrm{GO}_{poss} \left[ \begin{array}{ll} \mathrm{OBJECT} & \textit{money} \\ \mathrm{PATH} & \left[ \begin{array}{ll} \mathrm{FROM} & \textit{John} \\ \mathrm{TO} & \textit{Mary} \end{array} \right] \end{array} \right] \right]$$

As shown above, the three sentences are structurally distinguished only by a *semantic field feature*, notated as a subscript on the domain-general function GO. The use of domain-general representations with semantic field annotations allows Jackendoff to state both domain-general and domain-specific rules of inference. An example of a domain-general rule is that an object which goes from A to B is subsequently located at B [Jackendoff, 1992]. An example of a domain-specific rule is that no two objects may share the same spatial location.

### 2.1.3 Conceptual Spaces

One issue shared by both Conceptual Metaphor theory and Jackendoff's Conceptual Structure is that they conflate a wide range of different domains into the single domain of 'states' (in Jackendoff's system this domain is referred to as *identificational*, following Gruber [1965]). This is problematic because different types of states have different mathematical structures.

Peter Gärdenfors has developed a theory of the mathematical structure of domains like temperature, colour and emotion [Gärdenfors, 2004, 2014]. In his approach, each domain is treated as a separate *conceptual space*. A conceptual space is a mathematical space in which nearness corresponds to conceptual similarity and regions (connected sets of points) correspond to concepts.

For example, the colour conceptual space is the product space of hue, saturation and brightness. Hue is a circular dimension, which cycles through the colours of the rainbow; saturation is a linear dimension ranging from grey (low-intensity) to colourful (high-intensity); and brightness is a linear dimension ranging from black to white. The set of colours can be visualized as a 'spindle' (see Figure 2.2), where the disk in the centre corresponds to the well-known colour disk.

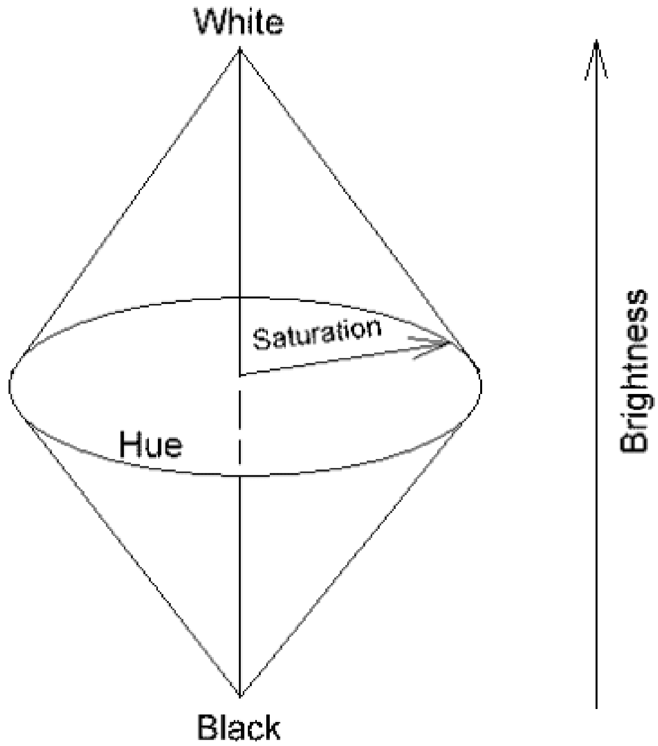


Figure 2.2: The colour spindle, from Goguen [2006].

Colour concepts like *red*, *green* and *blue* do not refer to exact colour values, but rather to regions in this space. Moreover, these regions have the property of *convexity*, meaning that, for any two points in the region, the point between them must also be in the region. An example of a non-convex colour concept would be *bled*, meaning 'dark blue or light red'. Gärdenfors claims that convexity is crucial for learnability, and that natural languages do not contain adjectives like *bled* because they are unlearnable.

There is good evidence for the convexity of colour adjectives. Jäger [2010] tested the convexity hypothesis using data from the World Colour Survey [Cook et al., 2005], which covers 330 colour chips for 110 languages. He calculated the optimal linear separator for each pair of colour terms in each language. The mean proportion of colour chips which were not reclassified by the linear separator was 93.8 percent. This is strong support for the convexity hypothesis because taking the union of all linear separators for a given colour term results in a convex region.

Conceptual spaces need not have a full metric structure. The only requirement is that they have a concept of *betweenness* to support the notion of convex region. I argue that the concept of betweenness, when formalized as an order relation obeying certain axioms, plays a crucial role in connecting the theory of physical space to property spaces like temperature, colour and emotion.

## 2.2 Conceptual Blending

### 2.2.1 Introduction to conceptual blending

Conceptual blending is a theory of analogy proposed by Fauconnier and Turner [2008], which is based on Fauconnier's concept of a *mental space*[1] [Fauconnier, 1994]. A mental space is a model of a cognitive domain consisting of set of concepts and relations between concepts. For example, the HOUSE domain might be modelled by the mental space shown in Figure 2.3.

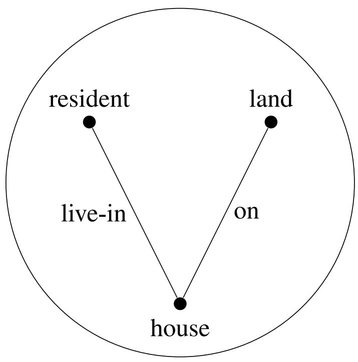


Figure 2.3: A mental space for the HOUSE domain. The space includes concepts *resident*, *land*, *house*, and relations *live-in*, *on*.

[1]Not to be confused with Gärdenfors' concept of a *conceptual space*, although the two ideas are very similar and can be unified by considering conceptual spaces to be a subset of mental spaces, namely those with the requisite topological/geometric structure.

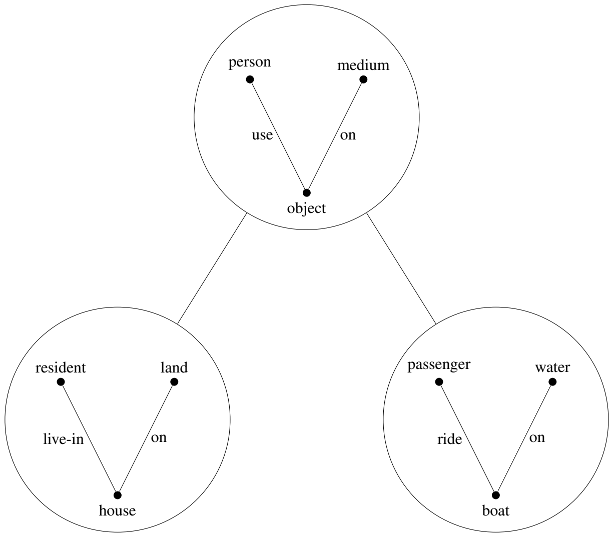


Figure 2.4: In conceptual blending, one first constructs a *generic space* which captures the common structure shared by the two input spaces.

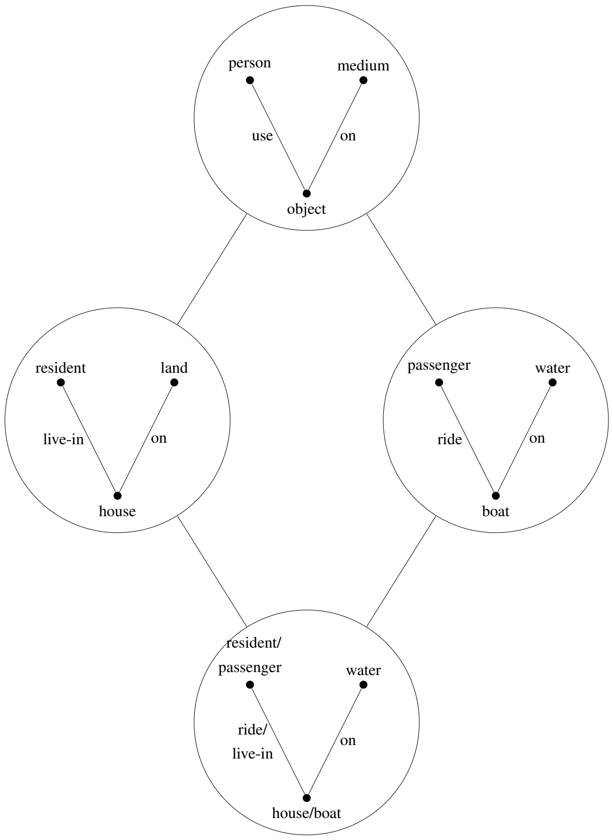


Figure 2.5: One then constructs a *blend space* which combines the two input spaces by preserving their common structure.

In the theory of conceptual blending, two mental spaces can be blended together. This is done by first constructing a third space, the *generic space*, which represents the structure shared by both input spaces. For example, the HOUSE space and the BOAT space share the concept of a person using an object, and an object being located on a medium (land in the case of the house, water in the case of the boat). This common structure gives rise to the generic space shown in Figure 2.4.

One then constructs a *blend space*, which incorporates information from both input spaces. This is done by requiring that the map from the generic space to the blend space is equivalent to the composite map via either input space. Figure 2.5 shows one possible blend of the house and boat spaces in which the blend space corresponds to the concept HOUSEBOAT. The blend space captures the fact that the resident of a houseboat is also its passenger.

### 2.2.2 Formalizing conceptual blending

This report follows the formalization of conceptual blending presented in Guhe et al. [2011]. In this approach, a mental space is represented as a theory in a many-sorted first order logic. Such a theory consists of two parts: (a) a *signature* specifying the sorts, predicates and functions used in the theory, and (b) a set of *axioms* which restrict the interpretation of the symbols in the signature. Links between theories are represented by *signature morphisms*, which map every symbol in the source signature onto some symbol in the target signature.

#### 2.2.2.1 Computing the generic space

The generic space in Guhe et al.'s approach is computing using Heuristic-Driven Theory Projection (HDTP). HDTP is a technique for generalizing first-order theories based on *anti-unification*, which is the formal counterpart to unification [Schwering et al., 2009, Gust et al., 2006]. In unification, two formulas are made equal by replacing variables with terms; in anti-unification two formula are made equal by replacing terms with variables.

First-order anti-unification allows replacements like those shown in Figure 2.6, where a single term is generalized to a variable. HDTP also uses a restricted form of higher-order anti-unification which allows variables to take arguments. This allows for substitutions in which functions are replaced by variables, as in Figure 2.7a. It is also possible to have substitutions where a function is replaced by a more complex structure involving embedded parts, as in Figure 2.7b.

HDTP computes the generalization of two input theories by iteratively choosing formulas from the two theories to be anti-unified. The anti-instances become the formulas of the new general theory, which describes the structural commonalities shared by the two input theories. The substitutions used to compute the anti-instances are preserved as theory morphisms mapping the symbols in generic space onto symbols in the two input spaces.

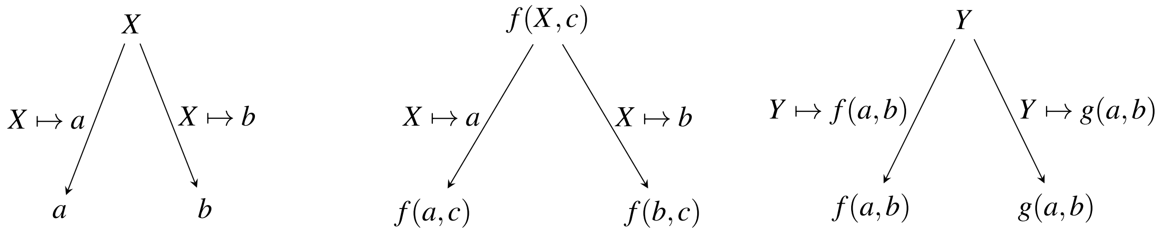


Figure 2.6: Examples of first-order anti-unification, from Schwering et al. [2009].

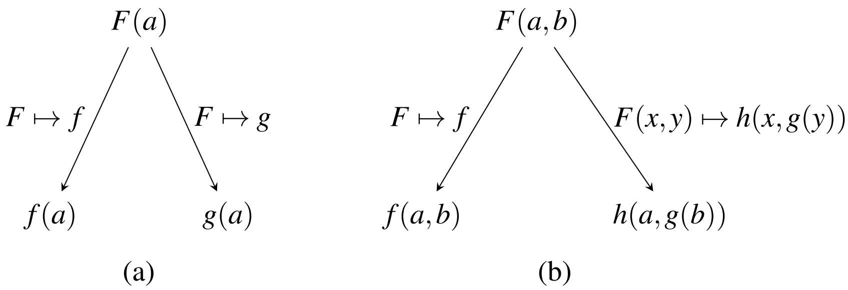


Figure 2.7: Examples of restricted higher-order anti-unification, from Schwering et al. [2009].

There are two main obstacles to this procedure. The first is that, for any formula in input space 1, there is a choice of formulas from input space 2 which can be anti-unified with it. The second is that, for any pair of formulas, there are several possible least-general anti-instances (although only finitely many due to the restrictions HDTP places on higher-order anti-unification [Gust et al., 2006]). HDTP tackles these problems by applying the following heuristics:

1. The order in which formulas from the two input theories are anti-unified depends on their structural complexity: structurally simpler formulas are aligned first.

2. Substitutions which can be reused across different pairs are preferred to novel substitutions. A substitution which has already been used can be re-used without additional cost.

3. Simpler substitutions are preferred to complex substitutions. Use of higher-order rather than first-order substitutions increases the cost.

There is no requirement that the generic space captures all the structure in both input theories. A formula in one of the input theories may have no correspondent in the other, in which case it will not be generalized to the generic space. Formulae which are not covered by the generic space are particularly important, since they provide domain-specific information to the blend.

#### 2.2.2.2 Computing the blend space

Once the generic space has been constructed, a blend space is found by first computing the *pushout* of the two substitution maps linking the generic space to the two input spaces. The pushout is the space $P$ such that the map from the generic space to $P$ is equivalent to the composite map via either of the input spaces - one can ‘go either way’ in the diagram. See Figure 2.8 for a formal definition.

$$
\begin{tikzcd}
& G \arrow[dl, "r_1"'] \arrow[dr, "r_2"] & \\
I_1 \arrow[dr, "s_1"'] & & I_2 \arrow[dl, "s_2"] \\
& P &
\end{tikzcd}
$$

Figure 2.8: Given a generic space $G$, input spaces $I_1, I_2$, and maps $r_1 : G \to I_1$, $r_2 : G \to I_2$, the pushout consists of the space $P$ and maps $s_1 : I_1 \to P$, $s_2 : I_2 \to P$, such that $s_1 \circ r_1 = s_2 \circ r_2$.

The substitution maps linking the generalization to the two input theories naturally extend to formulas. Formulas which contain symbols completely covered by the generalization will be imported into the blend in accordance with the pushout construction. The blend space preserves inferences, because any formulas which are proveable in either input theory are also proveable in the blend.

In addition to formulas covered by the generalization, all formulas not covered by the generalization are also imported into the blend. For example, if the HOUSE theory contained the additional information that a house can have a mortgage, then this can be imported into the HOUSEBOAT blend, even though it is not present in the BOAT space. This is generally a good thing, because it allows domain-specific knowledge in the two input theories to inform the blend.

However, importing information not covered by the generic space can often cause problems. If the two generic spaces contain conflicting information, then the resulting blend will be inconsistent. Even if the information which gets imported is not conflicting, it is sometimes too domain-specific to be relevant to the blend. For example, if the HOUSE domain contains the information that houses are made of bricks, then this will also be imported into the blend space, where it is clearly inappropriate. This problem is discussed further in Section 5.3.

# Chapter 3

# Formalizing Domains

It is important to distinguish between *scientific* theories and *common-sense* theories when formalizing knowledge. For example, in a common-sense theory of events, two events are either simultaneous or not simultaneous. However, in a scientific theory of events, this is only approximately true for objects which are near each other and moving slowly compared to the speed of light. The theories in this section are common-sense theories, which are intended to represent the understanding of a prototypical language learner.

I do not aim for mathematical exactness or elegance. Many mathematical theories, such as the theory of real numbers, are highly complex, and it is far from clear what role, if any, they play in a learner's understanding of space and quantity. For this reason, I leave most of the underlying set theory unspecified, and avoid set-theoretic topology in favour of order theory, since orders often provide a very intuitive formalization of conceptual domains.

Domains are formalizing using the Heterogeneous Tool Set (Hets) [Mossakowski et al., 2007]. Hets is a parsing, static analysis and proof management tool supporting a wide range of different specification languages. Libraries of specifications are represented using *development graphs*, in which nodes correspond to specifications and arrows show how specifications are related to each other. Its focus on the relations between specifications makes Hets particularly suited to setting up and studying blends.

Domains are written in the Common Algebraic Specification Language (CASL) Mosses [2004], a general-purpose specification language based on many-sorted first-order logic.[1]

## 3.1 Change in Place

The theory of spatial reasoning developed in this section is based on the theory of space in Davis' *Representations of Commonsense Knowledge* [Davis, 2014]. It contains six types of

[1] The domains formalized in this chapter are also included in the attached file `blends.casl`, and can be found online in the Ontohub repository at: `https://ontohub.org/dan-examples`.

entities: physical objects, points, distances, regions, times and fluents, which are represented by the sorts $Obj$, $Point$, $Dist$, $Region$, $Time$ and $Fluent$ respectively.

Mathematically, physical space is a *metric space*, meaning that for any two points, there is an associated real number which measures the distance between them. However, to keep the theory concise and intuitive, we can introduce an additional sort $Dist$, which represents separations between points. $Dist$ has those properties of real numbers which are most needed for a concept of distance, which are the total order given by $<$ and the concept of addition. The order structure and the algebraic structure are compatible. The minimum element 0 is also the additive identity.

$\forall\ x, y, z : Dist$
- $x < y \Rightarrow \neg\ y < x$ %(antisymmetry)%
- $x < y \wedge y < z \Rightarrow x < z$ %(transitivity)%
- $x < y \vee y < x \vee x = y$ %(totality)%
- $0 < x \vee x = 0$ %(minimum element)%
- $x + (y + z) = (x + y) + z$ %(associativity)%
- $x + 0 = x$ %(identity element)%
- $x + y = y + x$ %(commutativity)%
- $x < y \Rightarrow x + z < y + z$ %(order preservation)%

The function $dist : Point \times Point \rightarrow Dist$ takes two points and returns the distance between them. This function is required to satisfy the metric axioms. The axiom of non-negativity is automatically ensured because we have set up distances to always be greater than 0.

$\forall\ x, y, z : Point$
- $dist(x, x) = 0$ %(indiscernability)%
- $dist(x, y) = dist(y, x)$ %(symmetry)%
- $dist(x, z) < dist(x, y) + dist(y, z)$
  $\vee\ dist(x, z) = dist(x, y) + dist(y, z)$ %(triangle inequality)%

In physical space, we can say that a point $z$ is between points $x$ and $y$ (written $[x\ y\ z]$) iff $dist(x,z) = dist(x,y) + dist(y,z)$. In other words, $[x\ y\ z]$ is satisfied iff $y$ lies somewhere on the line segment $xz$. It follows from the distance axioms that the relation of betweenness defined this way is symmetric, anti-cyclic, and transitive. These properties are common to all betweenness relations (see Huntington [1935] for a list of betweenness axioms).

$\forall\ x, y, z, w : Point$
- $[x\ y\ z] \Leftrightarrow dist(x, z) = dist(x, y) + dist(y, z)$ %(definition of between)%
- $[x\ y\ z] \Rightarrow [z\ y\ x]$ %(symmetry)%

- $[x\ y\ z] \wedge [y\ z\ x] \Rightarrow x = y \wedge y = z$ %(anticyclicity)%
- $[x\ y\ z] \wedge [x\ z\ w] \Rightarrow [x\ y\ w]$ %(transitivity)%

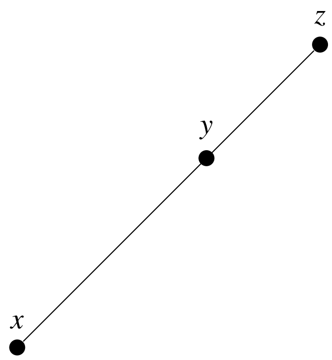


Figure 3.1: A model of $[x\ y\ z]$ in the spatial domain.

The sort *Region* represents regions of space. Regions are intended to refer to sets which are open and connected, but these properties are not formalized explicitly. Instead, we can make do with the most basic fact about regions, which is that two regions with the same members are equal. The subset relation $\subseteq$ between regions is defined in the usual way.

$\forall\ x, y : Region; z : Point$

- $x = y \Leftrightarrow (z \in x \Leftrightarrow z \in y)$ %(extensionality)%
- $x \subseteq y \Leftrightarrow z \in x \Rightarrow z \in y$ %(definition of subset)%

For the CHANGE IN PLACE domain to have a concept of movement, the structure of time also needs to be represented. Times are totally ordered by the 'precedes or simultaneous' relation. Time is assumed to be unbounded in both the past and future directions.

$\forall\ x, y, z : Time$

- $x \leq y \wedge y \leq x \Rightarrow y = x$ %(antisymmetry)%
- $x \leq y \wedge y \leq z \Rightarrow x \leq z$ %(transitivity)%
- $x \leq y \vee y \leq x$ %(totality)%
- $x < y \Leftrightarrow x \leq y \wedge \neg\ x = y$ %(definition of $<$)%

At any moment of time, a physical object occupies a region of space. However, for the purposes of this analysis, physical objects are assumed to have an identifiable coordinate system. This means that the location of an object can be given by a function from objects and times to points, which specifies the location of the object's coordinate system.

It is also convenient to *reify* the location function so that it can be quantified and used as an argument to predicates. A reified function of time is often called a *fluent* in the artificial intelligence literature, and is represented by the sort $Fluent$. Fluents are commonly used in formalisms which are specifically designed to represent change over time, such as the *situation calculus* [Levesque et al., 1998] and the *fluent calculus* [Thielscher, 1998].

Each object has an associated location fluent given by the (genuine) function $place : Obj \to Fluent$. The value of a fluent at a given time is given by the function $value\_at : Fluent \times Time \to Point$. Fluents are extensional, meaning that two fluents which agree on all times are equal. One advantage of using fluents is that one can directly state the fact that an object's location changes continuously using a single predicate *continuous*, without requiring a theory of topology[2] [3].

$$\forall\, x, y : Fluent;\ t : Time$$
$$\bullet\ x = y \Leftrightarrow value\_at(x, t) = value\_at(y, t) \qquad \%(\text{fluents are extensional})\%$$

$$\forall\, x : Obj$$
$$\bullet\ continuous(place(x)) \qquad \%(\text{objects move continuously})\%$$

In addition to its core axioms, the CHANGE IN PLACE domain contains concepts like *in*, *at*, *go*, *stay*, etc., which correspond to the meanings of words. See Section 4.1.3 for a formalization of some of these concepts.

See Appendix A.1 for the full theory of CHANGE IN PLACE. The syntax of the theory given in the appendix has been altered so as to be compatible with HDTP. Each statement is written on a separate line. Each axiom is preceded by a declaration of bound variables. Finally, all function and predicate symbols are renamed so as to be in prefix notation. The following substitutions are made: $\_\_ \leq \_\_ : Dist \times Dist \mapsto smaller$; $\_\_ + \_\_ \mapsto plus$; $[\_\_\_\_\_\_] \mapsto betw$; $\_\_ \in \_\_ \mapsto is\_in$; $\_\_ \subseteq \_\_ \mapsto subset$; $\_\_ \leq \_\_ : Time \times Time \mapsto precedes$; $\_\_ < \_\_ : Time \times Time \mapsto precedes\_neq$.

## 3.2 Change in Temperature

The domain of CHANGE IN TEMPERATURE is simpler than CHANGE IN PLACE and consists of only five types of entities: physical objects, temperatures, temperature regions, times and fluents. These are represented by the sorts *Obj*, *Temp*, *TempRegion*, *Time* and *Fluent* respectively.

The sort *Temp* represents exact temperatures, such as 0°C and 35°C, which are analogous to points in the physical space domain. However, unlike physical space, I do not assume that

[2] If a definition of *continuous* is required, then it can developed along the following lines. Define an open time interval $(x,y)$ to be the set of times $t$ such that $x < t < y$. Define an open ball $B(x,r)$ to be the set of points $p$ such that $dist(x,p) < r$. A set of times $V$ is a *neighbourhood* of a time $t$ if there is some open time interval $I$ such that $t \in I \subseteq V$. Similarly, a set of points $V$ is a *neighbourhood* of a point $p$ if there is some open ball $B$ such that $p \in B \subseteq V$. The predicate $continuous(f)$ requires that for all times $t$ and for all neighbourhoods $V_1$ of the point $value\_at(f,t)$, there is a neighbourhood $V_2$ of $t$ such that $x \in V_2 \Rightarrow value\_at(f,t) \in V_1$.

[3] This approach to reasoning about continuity is based on Davis [2014].

temperatures have an exact concept of distance. Rather, the structure of temperature is given by the ‘colder than or equal to’ relation, which is a total order.

$\forall\, x, y, z : \mathit{Temp}$
$\bullet\ x \leq y \wedge y \leq x \Rightarrow y = x$ %(antisymmetry)%
$\bullet\ x \leq y \wedge y \leq z \Rightarrow x \leq z$ %(transitivity)%
$\bullet\ x \leq y \vee y \leq x$ %(totality)%

In the CHANGE IN TEMPERATURE domain, a temperature $y$ is between temperatures $x$ and $z$ iff $x \leq y \leq z$ or $z \leq y \leq x$. It can be shown from the total order axioms that this relation satisfies the properties of a betweenness relation. Unlike betweenness for spatial points, betweenness for temperatures is also total.

$\forall\, x, y, z, w : \mathit{Temp}$
$\bullet\ [x\ y\ z] \Leftrightarrow (x \leq y \wedge y \leq z) \vee (z \leq y \wedge y \leq z)$ %(definition of between)%
$\bullet\ [x\ y\ z] \Rightarrow [z\ y\ x]$ %(symmetry)%
$\bullet\ [x\ y\ z] \wedge [y\ z\ x] \Rightarrow x = y \wedge y = z$ %(anticyclicity)%
$\bullet\ [x\ y\ z] \wedge [x\ z\ w] \Rightarrow [x\ y\ w]$ %(transitivity)%
$\bullet\ [x\ y\ z] \vee [y\ z\ x] \vee [z\ x\ y]$ %(totality)%

The CHANGE IN TEMPERATURE domain contains the concept of a temperature region, which is a set of exact temperature values. The axioms for temperature regions are exactly analogous to spatial regions.

$\forall\, x, y : \mathit{TempRegion};\ z, v, w : \mathit{Temp}$
$\bullet\ x = y \Leftrightarrow (z \in x \Leftrightarrow z \in y)$ %(extensionality)%
$\bullet\ x \subseteq y \Leftrightarrow z \in x \Rightarrow z \in y$ %(definition of subset)%

Following Gärdenfors, temperature adjectives like *cold*, *cool*, *warm*, *hot*, etc., do not refer to precise temperature values but rather to convex regions in temperature space. If a temperature $x$ can be described as *warm* and another temperature $y$ can be described as *warm*, then any temperature in-between can also be described as *warm*. Because temperature is a one-dimensional domain, the statement that regions are convex is equivalent to requiring regions to be intervals. These intervals can overlap, as for *tepid* and *warm*, and are not necessarily bounded: for example, *hot* and *cold* both refer to unbounded rays.

Like the CHANGE IN PLACE domain, the CHANGE IN TEMPERATURE domain contains a full set of time axioms. These are necessary in order to properly represent change.

$\forall\, x, y, z : \mathit{Time}$
$\bullet\ x \leq y \wedge y \leq x \Rightarrow y = x$ %(antisymmetry)%

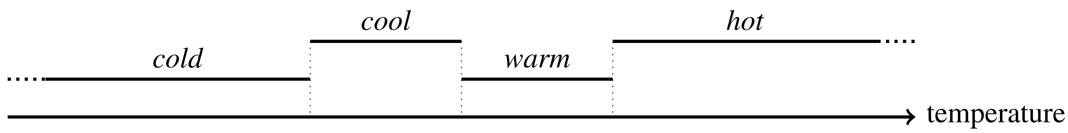


Figure 3.2: Temperature adjectives refer to convex regions in temperature space.

- $x \leq y \wedge y \leq z \Rightarrow x \leq z$ %(transitivity)%
- $x \leq y \vee y \leq x$ %(totality)%
- $x < y \Leftrightarrow x \leq y \wedge \neg\, x = y$ %(definition of <)%

In the CHANGE IN TEMPERATURE domain, fluents refer to functions from times to temperatures. There is one such fluent associated with each object, given by the function $temp : Obj \to Fluent$. The value of a fluent at a particular time is given by $value\_at : Fluent \times Time \to Temp$. As in the CHANGE IN PLACE domain, fluents in the CHANGE IN TEMPERATURE domain are continuous - an object's temperature does not jump from one value to another without passing through all values in-between.

$\forall\, x, y : \mathit{Fluent}; t : \mathit{Time}$
- $x = y \Leftrightarrow value\_at(x, t) = value\_at(y, t)$ %(fluents are extensional)%

$\forall\, x : \mathit{Obj}$
- $continuous(temp(x))$ %(objects change continuously)%

For the full theory of CHANGE IN TEMPERATURE, see Appendix A.2. as before, the version of CHANGE IN TEMPERATURE given in the appendix has been made compatible with HDTP. The following symbols are renamed: $\_\_ \leq \_\_ : Temp \times Temp \mapsto colder$; $[\_\_,\_\_] \mapsto betw$; $\_\_ \in \_\_ \mapsto is\_in$; $\_\_ \subseteq \_\_ \mapsto subset$; $\_\_ \leq \_\_ : Time \times Time \mapsto precedes$; $\_\_ < \_\_ : Time \times Time \mapsto precedes\_neq$.

## 3.3 Change in Emotion

The domain of CHANGE IN EMOTION contains seven types of entities: people, valence values, arousal values, emotions, emotion regions, times and fluents, represented by the sorts *Person*, *Val*, *Arous*, *Emot*, *EmotRegion*, *Time* and *Fluent* respectively.

There are several different theories of the structure of emotion. Most researchers agree that emotions comprise a continuous domain rather than a collection of discrete states (but see Izard et al. [1993] for a defence of discrete emotions). However, the mathematical structure of this space is the subject of some debate.

Prominent theories include Russell's circumplex model [Posner et al., 2005], in which emotions are cyclically ordered; the vector model of emotions [Bradley et al., 1992], where emotions form a two-dimensional vector space; the Pleasure-Arousal-Dominance (PAD) model [Mehrabian, 1980], where they form a three-dimensional vector space; and Pluchick's hybrid model [Plutchik, 2001], where there is one cyclic dimension and one linear dimension. For an empirical comparison of different dimensional models of emotion, see Rubin and Talarico [2009].

The majority of theories of emotion recognise at least the dimensions of *valence* and *arousal*. Valence is the enjoyableness of the emotion and ranges from highly pleasant to highly unpleasant. Arousal is the 'energy' of the emotion and ranges from calm to highly excited emotional states. Following Gärdenfors [2014], I analyse emotion as the two-dimensional product space of valence and arousal (see Figure 3.3).

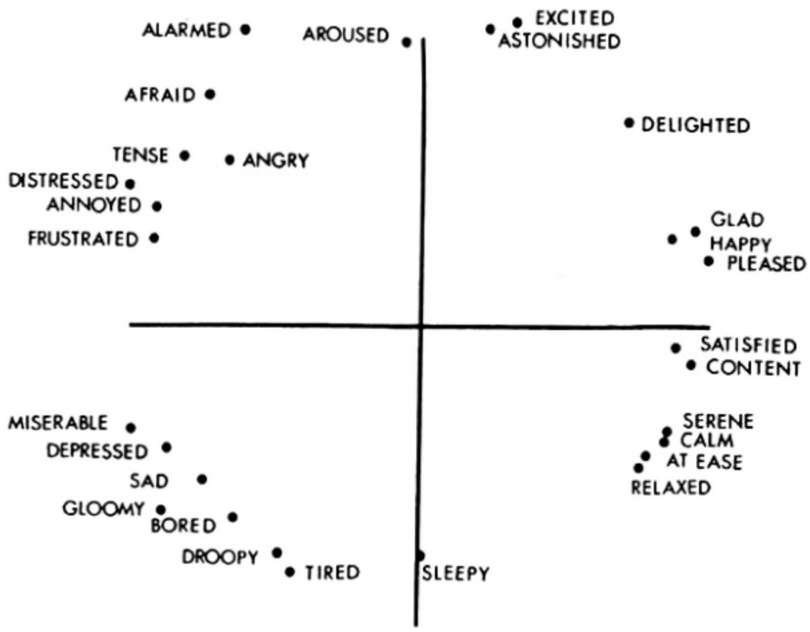


Figure 3.3: A two-dimensional emotion space, from Posner et al. [2005]. Valence is plotted horizontally, arousal is plotted vertically.

Valence and arousal values are represented by the sorts *Val* and *Arous* respectively. Valence and arousal are both totally ordered linear domains which only vary within a limited range of values. The upper and lower bounds are represented by the individuals *min_val*, *max_val*, *min_arous* and *max_arous*.

$\forall\, x, y, z : Val$

$\bullet\; x \leq y \wedge y \leq x \Rightarrow y = x$ %(antisymmetry)%

$\bullet\; x \leq y \wedge y \leq z \Rightarrow x \leq z$ %(transitivity)%

$\bullet\; x \leq y \vee y \leq x$ %(totality)%

- $min\_val \leq x$ %(minimum element)%
- $x \leq max\_val$ %(maximum element)%
- $[x\ y\ z] \Leftrightarrow (x \leq y \wedge y \leq z) \vee (z \leq y \wedge y \leq z)$ %(definition of between)%

$\forall\ x, y, z : Arous$

- $x \leq y \wedge y \leq x \Rightarrow y = x$ %(antisymmetry)%
- $x \leq y \wedge y \leq z \Rightarrow x \leq z$ %(transitivity)%
- $x \leq y \vee y \leq x$ %(totality)%
- $min\_arous \leq x$ %(minimum element)%
- $x \leq max\_arous$ %(maximum element)%
- $[x\ y\ z] \Leftrightarrow (x \leq y \wedge y \leq z) \vee (z \leq y \wedge y \leq z)$ %(definition of between)%

The space of emotion values should be thought of as the product space of valence and arousal. To encode this, we can introduce projection functions $val : Emot \rightarrow Val$ and $arous : Emot \rightarrow Arous$, which take an emotion and return its valence and arousal values respectively. We also need an equality axiom for emotions, which states that two emotions are the same if and only if their valence and arousal values are the same.

$\forall\ x, y : Emot$

- $x = y \Leftrightarrow val(x) = val(y) \wedge arous(x) = arous(y)$ %(equality for emotions)%

There is also a notion of betweenness for emotions, which arises naturally from its product structure. An emotion $y$ is between two emotions $x$ and $z$ if $val(y)$ is between $val(x)$ and $val(z)$, and $arous(y)$ is between $arous(x)$ and $arous(z)$, where betweenness in the valence and arousal dimensions is understood in the same way as temperature (see Figure 3.4 for an illustration). It can be shown from the axioms for valence and arousal that this relation has all the properties of a betweenness relation.

$\forall\ x, y, z, w : Emot$

- $[x\ y\ z] \Leftrightarrow [val(x)\ val(y)\ val(z)]$
  $\wedge\ [arous(x)\ arous(y)\ arous(z)]$ %(definition of between)%
- $[x\ y\ z] \Rightarrow [z\ y\ x]$ %(symmetry)%
- $[x\ y\ z] \wedge [y\ z\ x] \Rightarrow x = y \wedge y = z$ %(anticyclicity)%
- $[x\ y\ z] \wedge [x\ z\ w] \Rightarrow [x\ y\ w]$ %(transitivity)%

The CHANGE IN EMOTION domain also contains a concept of an emotion region, which is a set of exact emotion values. The axioms for emotion regions are analogous to those for spatial regions.

$\forall\ x, y : EmotRegion;\ z, v, w : Emot$

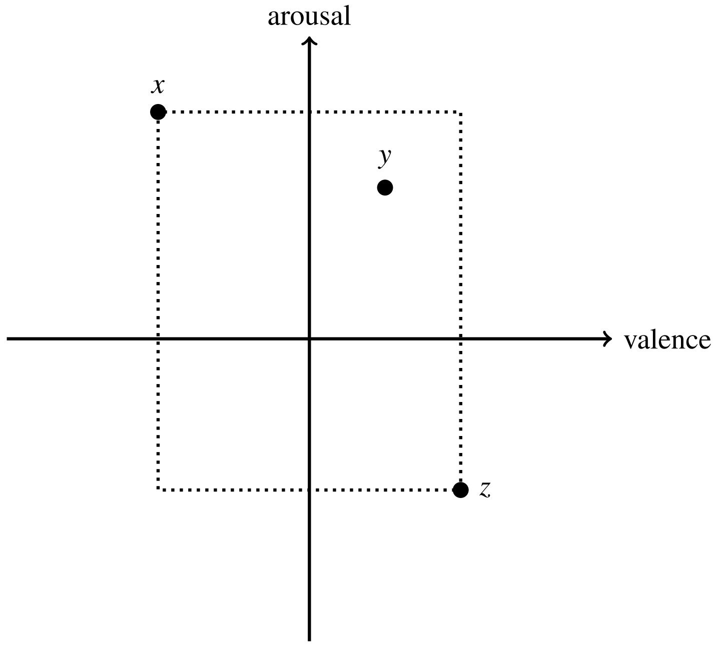


Figure 3.4: A model of $[x\ y\ z]$ in the emotion domain.

- $x = y \Leftrightarrow (z \in x \Leftrightarrow z \in y)$ %(extensionality)%
- $x \subseteq y \Leftrightarrow z \in x \Rightarrow z \in y$ %(definition of subset)%

Like temperature adjectives, emotion adjectives such as *happy*, *sad*, *angry* and *calm* refer to regions of emotion space rather than precise emotion values. Moreover, following Gärdenfors, these regions have the property of *convexity*. If an emotion $x$ can be described as *angry* and another emotion $y$ can be described as *angry*, then any emotion in-between can also be described as *angry*. Convex regions in emotion space are products of valence intervals and arousal intervals.

Like space and temperature, the CHANGE IN EMOTION domain contains axioms for times.

$\forall\, x, y, z : Time$

- $x \leq y \land y \leq x \Rightarrow y = x$ %(antisymmetry)%
- $x \leq y \land y \leq z \Rightarrow x \leq z$ %(transitivity)%
- $x \leq y \lor y \leq x$ %(totality)%
- $x < y \Leftrightarrow x \leq y \land \neg\, x = y$ %(definition of <)%

There is also a notion of fluent in the CHANGE IN EMOTION domain. The emotion fluent associated with a person is given by the function $emot : Person \rightarrow Fluent$. The value of a fluent at a specific time is given by the function $value\_at : Fluent \times Time \rightarrow Emot$. As in the CHANGE IN PLACE and CHANGE IN TEMPERATURE domains, fluents are extensional. However, emotion fluents are not required to be continuous because intuitively a person's mood can jump from one value to another without passing through intermediate moods.

$\forall\, x,\, y : Fluent;\, t : Time$

$\bullet\; x = y \Leftrightarrow value\_at(x,\, t) = value\_at(y,\, t)$ %(fluents are extensional)%

For the full theory of CHANGE IN EMOTION, see Appendix A.3. The following symbols are renamed in the appendix: $\_\_ \leq \_\_ : Val \times Val \mapsto better$; $[\_\_\_\_\_\_] : Val \times Val \times Val \mapsto betw\_val$; $\_\_ \leq \_\_ : Arous \times Arous \mapsto calmer$; $[\_\_\_\_\_\_] : Arous \times Arous \times Arous \mapsto betw\_arous$; $[\_\_\_\_\_\_] : Emot \times Emot \times Emot \mapsto betw$; $\_\_ \in \_\_ \mapsto is\_in$, $\_\_ \subseteq \_\_ \mapsto subset$; $\_\_ \leq \_\_ : Time \times Time \mapsto precedes$, $\_\_ < \_\_ : Time \times Time \mapsto precedes\_neq$.

## 3.4 Inheritance

The domain of inheritance (in the sense of *John inherited the money*) is interesting because it shows how the structure of betweenness extends to partially ordered domains. The INHERITANCE domain contains five types of entities: people, sets of people, heritable objects, times and fluents, which are represented by the sorts *Person*, *Set*, *Heritable*, *Time* and *Fluent* respectively.

People are ordered by ancestry, which can be axiomatized in several ways. One way is to define the ancestor relation recursively as the transitive closure of the parenthood relation (this approach is taken by Davis [2014]). However, for the purpose of this report it is convenient to work with the ancestor relation directly and leave parenthood implicit[4]. Ancestry has the properties of a partial order. In order to have a non-strict partial order, we must assume that everyone is, in a trivial sense, their own ancestor.

$\forall\, x,\, y,\, z : Person$

$\bullet\; ancestor(x,\, x)$ %(reflexivity)%

$\bullet\; ancestor(x,\, y) \wedge ancestor(y,\, x) \Rightarrow x = y$ %(antisymmetry)%

$\bullet\; ancestor(x,\, y) \wedge ancestor(y,\, z) \Rightarrow ancestor(x,\, z)$ %(transitivity)%

In the INHERITANCE domain, a person $y$ is between person $x$ and person $z$ iff $x$ is an ancestor of $y$ who is an ancestor of $z$, or $z$ is an ancestor of $y$ who is an ancestor of $z$. It can be shown from the partial order axioms that this relation obeys the betweenness properties. See Figure 3.5 for an illustration of this relation.

$\forall\, x,\, y,\, z,\, w : Person$

$\bullet\; [xyz] \Leftrightarrow (ancestor(x,\, y) \wedge ancestor(y,\, z))$

$\qquad \vee\, (ancestor(z,\, y) \wedge ancestor(y,\, x))$ %(definition of between)%

$\bullet\; [x\; y\; z] \Rightarrow [z\; y\; x]$ %(symmetry)%

[4]Alternatively, parenthood can be defined as follows: $parent(x,y) \Leftrightarrow ancestor(x,y) \wedge x \neq y \wedge \neg\exists z\, .\, ancestor(x,z) \wedge ancestor(z,y)$

- $[x\ y\ z] \wedge [y\ z\ x] \Rightarrow x = y \wedge y = z$ %(anticyclicity)%
- $[x\ y\ z] \wedge [x\ z\ w] \Rightarrow [x\ y\ w]$ %(transitivity)%

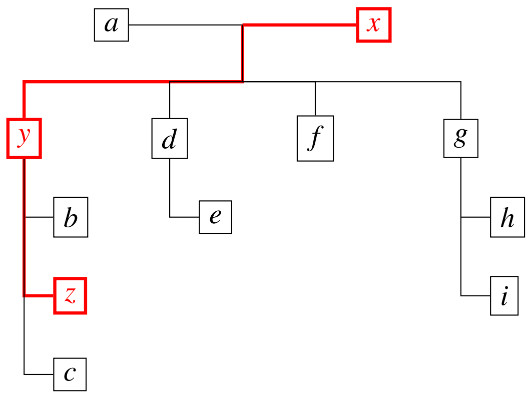


Figure 3.5: A model of $[x\ y\ z]$ in the INHERITANCE domain (shown in red).

The analogue of a region in the INHERITANCE domain is a set of people. As for regions, sets of people obey the axiom of extensionality.

$\forall\, x, y : \mathit{Set};\ z, v, w : \mathit{Person}$
- $x = y \Leftrightarrow (z \in x \Leftrightarrow z \in y)$ %(extensionality)%
- $x \subseteq y \Leftrightarrow z \in x \Rightarrow z \in y$ %(definition of subset)%

Convex regions in the INHERITANCE domain do not correspond to *families*, but to *lines of descent* or *lineages*, as in *"She could trace her lineage back to King Alfred"*. It is possible to have a conceptual space in which families correspond to convex regions, but this requires a space with a different mathematical structure (see Gärdenfors [2004]). The convex regions in the INHERITANCE domain are the same as the intervals given by the partial order.

In order to represent change, the INHERITANCE domain must contain the time axioms:

$\forall\, x, y, z : \mathit{Time}$
- $x \leq y \wedge y \leq x \Rightarrow y = x$ %(antisymmetry)%
- $x \leq y \wedge y \leq z \Rightarrow x \leq z$ %(transitivity)%
- $x \leq y \vee y \leq x$ %(totality)%
- $x < y \Leftrightarrow x \leq y \wedge \neg\, x = y$ %(definition of <)%

Change in the INHERITANCE domain means the change in ownership of heritable entities. Heritable entities can be objects, such as family heirlooms; locations, such as areas of land; or titles, such as King of England. All of these are represented by the sort *Heritable*. The time-dependent owner of a heritable object is given by the function $\mathit{owner} : \mathit{Heritable} \rightarrow \mathit{Fluent}$. The value of the fluent at a specific time is given by $\mathit{value\_at} : \mathit{Fluent} \times \mathit{Time} \rightarrow \mathit{Person}$.

There are two constraints on how heritable objects can change owner. The first is that they move continuously, respecting the topology associated with the partial order *ancestor*. The second is that they only move down the ancestor tree, never up - one does not inherit something from one's descendants.

$\forall\, x, y : \mathit{Fluent};\ t : \mathit{Time}$
$\bullet\ x = y \Leftrightarrow \mathit{value\_at}(x, t) = \mathit{value\_at}(y, t)$ %(fluents are extensional)%

$\forall\, x : \mathit{Heritable}$
$\bullet\ \mathit{continuous}(\mathit{owner}(x))$ %(inheritance is continuous)%

$\forall\, x : \mathit{Fluent};\ \mathit{t1}, \mathit{t2} : \mathit{Time}$
$\bullet\ \mathit{t1} < \mathit{t2} \Rightarrow \mathit{ancestor}(\mathit{value\_at}(x, \mathit{t1}), \mathit{value\_at}(x, \mathit{t2}))$ %(inheritance is downward)%

For the complete theory of INHERITANCE, see Appendix A.4. The following symbols are renamed in the appendix: $[\_\_\_\_\_\_] : \mathit{Person} \times \mathit{Person} \mapsto \mathit{betw}$; $\_\_ \in \_\_ \mapsto \mathit{is\_in}$; $\_\_ \subseteq \_\_ \mapsto \mathit{subset}$; $\_\_ \leq \_\_ \mapsto \mathit{precedes}$, $\_\_ < \_\_ \mapsto \mathit{precedes\_neq}$.

## 3.5 Summary

This chaper has formalized four domains, CHANGE IN PLACE, CHANGE IN TEMPERATURE, CHANGE IN EMOTION and INHERITANCE, which are intended to represent the understanding of a language learner. The next section blends CHANGE IN PLACE with the other three domains as a test of whether conceptual blending can model the transfer of spatial information into non-spatial domains.

# Chapter 4

# Experiments

This section presents a series of blending experiments, which are intended to test whether conceptual blending can model the transfer of spatial knowledge into other domains. There are three main experiments, which blend CHANGE IN PLACE with the theories of CHANGE IN TEMPERATURE, CHANGE IN EMOTION and INHERITANCE respectively. A fourth experiment blends CHANGE IN TEMPERATURE with CHANGE IN EMOTION for the sake of completeness and because this raises an interesting problem. I begin by outlining the evaluation criteria which I expect the blends to fulfil.

What might prompt a learner to construct the blends outlined in this section? It is not sufficient to say that the learner simply tries out all possible blends, since the number of possible blends is exponential in the amount of knowledge they have acquired, and it is unclear how the learner would know if he or she has discovered a useful blend.

Instead, I assume that the blending of knowledge is linguistically guided. For example, suppose a learner who only knows the meaning of *go* in the spatial domain, encounters the sentence *Your food is going cold*. This sentence prompts them to construct a blend of CHANGE IN PLACE and CHANGE IN TEMPERATURE, resulting in a theory which contains the predicate *go* alongside the physical object $food$ and the temperature region $cold$.

The blends in this section are computed using the COINVENT implementation of conceptual blending (see [Mohrmann et al.]), which is interfaced with Hets. The COINVENT project is aimed at developing a computationally-feasible, cognitively-inspired formal model of concept invention, drawing on the theory of conceptual blending and Goguen's Unified Concept Theory [Schorlemmer et al., 2014].

## 4.1 Evaluation Criteria

The following three criteria are used to evaluate the experiments:

1. The generic space should contain the theory CHANGE IN CONCEPTUAL SPACE, which

contains Gärdenfors' concept of a conceptual space (see Section 4.1.1 below). This counts as evidence for Research Hypothesis 3, that analogies between physical space and other domains include the conceptual space structure.

2. The blend space should be mathematically consistent. This counts as evidence for Research Hypothesis 1, that physical space can be consistently blended with other domains.

3. The blend space should contain the predicates *at*, *in*, $go_1$, $go_2$, $stay_1$, and $stay_2$ (see Section 4.1.4 below). This counts as evidence for Research Hypothesis 2, that blending can model the transfer of spatial concepts into new domains.

This section explains the above criteria in more detail.

### 4.1.1 Target generic space

The domains CHANGE IN PLACE, CHANGE IN TEMPERATURE, CHANGE IN EMOTION and INHERITANCE share a common structure. In addition to the concepts of region, time and fluent, all four theories contain the structure of a conceptual space in the sense of Gärdenfors [2004, 2014], which is a set structured by a betweenness relation. The common structure shared by all four theories is given by the the theory CHANGE IN CONCEPTUAL SPACE, shown below.

**spec** CHANGE_IN_CONCEPTUAL_SPACE =

**sorts** *Obj*, *State*, *Set*, *Time*, *Fluent*
**preds** $[\_\_\_]$ : $State \times State \times State$;
$\_\_\leq\_\_$ : $Time \times Time$;
$\_\_<\_\_$ : $Time \times Time$;
$\_\_\in\_\_$ : $State \times Set$;
$\_\_\subseteq\_\_$ : $Set \times Set$
**ops** $state : Obj \rightarrow Fluent$;
$value\_at : Fluent \times Time \rightarrow State$

%betweenness axioms%
$\forall\, x, y, z, w : State$
• $[x\ y\ z] \Rightarrow [z\ y\ x]$ %(symmetry)%
• $[x\ y\ z] \wedge [y\ z\ x] \Rightarrow x = y \wedge y = z$ %(anticyclicity)%
• $[x\ y\ z] \wedge [x\ z\ w] \Rightarrow [x\ y\ w]$ %(transitivity)%

%set axioms%
$\forall\, x, y : Set;\ z, v, w : State$

$\bullet\ x = y \Leftrightarrow (z \in x \Leftrightarrow z \in y)$ %(extensionality)%

$\bullet\ x \subseteq y \Leftrightarrow z \in x \Rightarrow z \in y$ %(definition of subset)%

%time axioms%

$\forall\ x, y, z : \mathit{Time}$

$\bullet\ x \leq y \wedge y \leq x \Rightarrow y = x$ %(antisymmetry)%

$\bullet\ x \leq y \wedge y \leq z \Rightarrow x \leq z$ %(transitivity)%

$\bullet\ x \leq y \vee y \leq x$ %(totality)%

$\bullet\ x < y \Leftrightarrow x \leq y \wedge \neg\, x = y$ %(definition of $<$)%

%fluent axioms%

$\forall\ x, y : \mathit{Fluent};\ t : \mathit{Time}$

$\bullet\ x = y \Leftrightarrow \mathit{value\_at}(x, t) = \mathit{value\_at}(y, t)$ %(fluents are extensional)%

**end**

The CHANGE IN CONCEPTUAL SPACE theory contains the abstract concept of a *State*, which is structured by a betweenness relation. *State* covers *Point* in the spatial domain, *Temp* in the temperature domain, *Emot* in the emotion domain, and *Person* in the ancestry domain. There is also a concept of a *Set* of states, which covers *Region* in the spatial domain, *TempRegion* in the temperature domain, *EmotRegion* in the emotion domain, and *Set* in the ancestry domain.

The inclusion morphisms linking CHANGE IN CONCEPTUAL SPACE to CHANGE IN PLACE, CHANGE IN TEMPERATURE, CHANGE IN EMOTION and INHERITANCE are shown below:

**view** CONCEPTUAL_TO_SPACE :

CHANGE_IN_CONCEPTUAL_SPACE **to** CHANGE_IN_PLACE =

$\mathit{Obj} \mapsto \mathit{Obj}$, $\mathit{State} \mapsto \mathit{Point}$, $\mathit{Set} \mapsto \mathit{Region}$, $\mathit{Time} \mapsto \mathit{Time}$,

$\mathit{Fluent} \mapsto \mathit{Fluent}$, $[\_\_\_\_\_\_] \mapsto [\_\_\_\_\_\_]$,

$\_\_\leq\_\_ \mapsto \_\_\leq\_\_$, $\_\_<\_\_ \mapsto \_\_<\_\_$,

$\_\_\in\_\_ \mapsto \_\_\in\_\_$, $\_\_\subseteq\_\_ \mapsto \_\_\subseteq\_\_$,

$\mathit{state} \mapsto \mathit{place}$, $\mathit{value\_at} \mapsto \mathit{value\_at}$

**end**

**view** CONCEPTUAL_TO_TEMPERATURE :

CHANGE_IN_CONCEPTUAL_SPACE **to** CHANGE_IN_TEMPERATURE =

$\mathit{Obj} \mapsto \mathit{Obj}$, $\mathit{State} \mapsto \mathit{Temp}$, $\mathit{Set} \mapsto \mathit{TempRegion}$, $\mathit{Time} \mapsto \mathit{Time}$,

$\mathit{Fluent} \mapsto \mathit{Fluent}$, $[\_\_\_\_\_\_] \mapsto [\_\_\_\_\_\_]$,

$\_\_\leq\_\_ \mapsto \_\_\leq\_\_$, $\_\_<\_\_ \mapsto \_\_<\_\_$,

$\_\_\in\_\_ \mapsto \_\_\in\_\_$, $\_\_\subseteq\_\_ \mapsto \_\_\subseteq\_\_$,

*state* $\mapsto$ *temp*, *value_at* $\mapsto$ *value_at*

**end**

**view** CONCEPTUAL_TO_EMOTION :

CHANGE_IN_CONCEPTUAL_SPACE **to** CHANGE_IN_EMOTION =

*Obj* $\mapsto$ *Person*, *State* $\mapsto$ *Emot*, *Set* $\mapsto$ *EmotRegion*,

*Time* $\mapsto$ *Time*, *Fluent* $\mapsto$ *Fluent*, $[\_\_\_\_\_\_] \mapsto [\_\_\_\_\_\_]$,

$\_\_\leq\_\_ \mapsto \_\_\leq\_\_$, $\_\_<\_\_ \mapsto \_\_<\_\_$,

$\_\_\in\_\_ \mapsto \_\_\in\_\_$, $\_\_\subseteq\_\_ \mapsto \_\_\subseteq\_\_$,

*state* $\mapsto$ *emot*, *value_at* $\mapsto$ *value_at*

**end**

**view** CONCEPTUAL_TO_ANCESTRY :

CHANGE_IN_CONCEPTUAL_SPACE **to** INHERITANCE =

*Obj* $\mapsto$ *Heritable*, *State* $\mapsto$ *Person*, *Set* $\mapsto$ *Set*, *Time* $\mapsto$ *Time*,

*Fluent* $\mapsto$ *Fluent*, $[\_\_\_\_\_\_] \mapsto [\_\_\_\_\_\_]$,

$\_\_\leq\_\_ \mapsto \_\_\leq\_\_$, $\_\_<\_\_ \mapsto \_\_<\_\_$,

$\_\_\in\_\_ \mapsto \_\_\in\_\_$, $\_\_\subseteq\_\_ \mapsto \_\_\subseteq\_\_$,

*state* $\mapsto$ *owner*, *value_at* $\mapsto$ *value_at*

**end**

For the blends to be successful, the generalization discovered by HDTP should include the theory CHANGE IN CONCEPTUAL SPACE. The substitution morphisms should resemble the inclusion morphisms given above. For example, when blending space and temperature, the algorithm should discover the analogy between points and temperatures, spatial regions and temperature regions, place fluents and temperature fluents, and so on.

The fact that some symbols from different theories have the same names, such as the sorts $Obj$, $Time$, $Fluent$, does not help the algorithm, because HDTP does not assume that identical symbols from the two input theories must be identified in the generic space. Instead, the algorithm discovers analogies solely on the basis of structural similarity between axioms.

### 4.1.2 Consistency of blend

The foremost constraint on blends is that they should be consistent. The most likely source of inconsistency is the betweenness relation $[x\ y\ z]$, because this is defined differently in all four domains. For the blends to be consistent, the definitions of $[x\ y\ z]$ in the two input spaces must be compatible.

For example, in the CHANGE IN PLACE domain, the expression $[x\,y\,z]$ is defined as $dist(x,z) = dist(x,y) + dist(y,z)$ ('$y$ lies on the line segment $xz$'). However, in the CHANGE IN TEMPERATURE domain, the expression $[x\,y\,z]$ is defined as $(x \leq y \leq z) \vee (z \leq y \leq x)$ ('$y$ is between $x$ and $z$ in temperature'). The two definitions must be compatible for the blend to be successful.

If the definitions of betweenness from the two input theories agree, then the blend space will have a single concept of convex set. This is desirable because, as explained in Section 2.1.3, convexity is an important property of adjectives. The location and shape of the regions *warm*, *hot*, *cold*, etc., should not change when CHANGE IN TEMPERATURE is blended with CHANGE IN PLACE.

### 4.1.3 Spatial concepts are transferred correctly

In addition to the axioms outlined in Section 3.1, the CHANGE IN PLACE domain contains concepts like *at*, *in*, *go* and *stay*, which correspond to the meanings of words. Following Conceptual Metaphor Theory [Lakoff and Núñez, 2000, Lakoff and Johnson, 2008], the meanings of these words are learned first in the context of physical space before being transferred to other domains.

One example of a lexicalized concept is location. I use the predicate *at* to mean location at a precise point, and *in* to mean location in a region, with the caveat that the English word *at* is actually used for both of these situations (*I'm at the supermarket* vs. *I'm at forty-one degrees north, twenty-eight degrees east*). This highlights the important point that there is rarely a one-to-one correspondence between words in a natural language and symbols in the formal language one is using to study its semantics.

$\forall\, x : Obj;\ y : Point;\ t : Time$

$\bullet\ at(x,\ y,\ t) \Leftrightarrow value\_at(place(x),\ t) = y$ %(definition of at)%

$\forall\, x : Obj;\ y,\ z : Region;\ t : Time$

$\bullet\ in(x,\ y,\ t) \Leftrightarrow value\_at(place(x),\ t) \in y$ %(definition of in)%

The meaning of the word *go* is represented by two predicates $go_1$ and $go_2$. The predicate $go_1$ corresponds to movement between precise points, whereas $go_2$ corresponds to movement between regions. The expression $go_1(x,y,z,t_1,t_2)$ means 'object $x$ goes from point $y$ at time $t_1$ to point $z$ at time $t_2$'. The expression $go_2(x,y,z,t_1,t_2)$ means 'object $x$ goes from region $y$ at time $t_1$ to region $z$ at time $t_2$'.

$\forall\, x : Obj;\ y,\ z : Point;\ t_1,\ t_2 : Time$

$\bullet\ go_1(x,\ y,\ z,\ t_1,\ t_2) \Leftrightarrow at(x,\ y,\ t_1) \wedge at(x,\ z,\ t_2) \wedge t_1 < t_2$ %(definition of go$_1$)%

$\forall\, x : Obj;\, y, z : Region;\, t_1, t_2 : Time$

$\bullet\; go_2(x, y, z, t_1, t_2) \Leftrightarrow in(x, y, t_1) \wedge in(x, z, t_2) \wedge t_1 < t_2$ %(definition of go$_2$)%

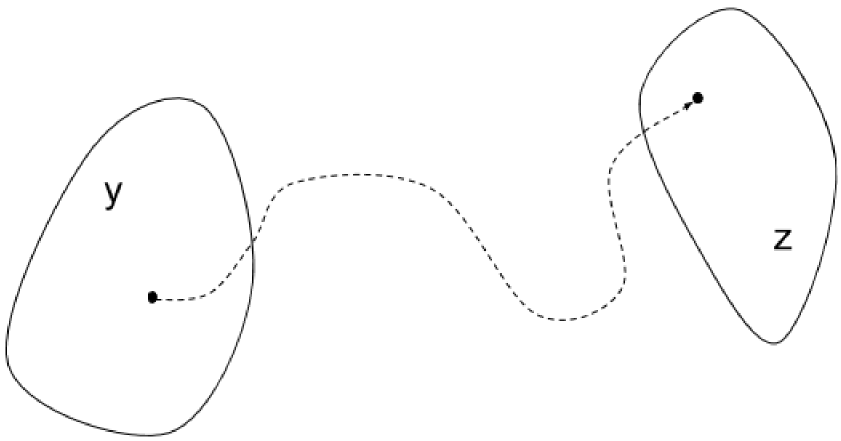


Figure 4.1: A model of $go_2$ in the spatial domain.

The concept *stay* is closely linked to *go*; like *go*, it also has two varieties $stay_1$ and $stay_2$ corresponding to exact locations and regions respectively. The expression $stay_1(x,y,t_1,t_2)$ means 'object $x$ stays at point $y$ from time $t_1$ to time $t_2$'. The expression $stay_1(x,y,t_1,t_2)$ means 'object $x$ stays in region $y$ from time $t_1$ to time $t_2$'.

$\forall\, x : Obj;\, y : Point;\, t_1, t_2, t_3 : Time$

$\bullet\; stay_1(x, y, t_1, t_3) \Leftrightarrow t_1 \le t_2 \wedge t_2 \le t_3 \Rightarrow at(x, y, t_2)$ %(definition of stay$_1$)%

$\forall\, x : Obj;\, y, z : Region;\, t_1, t_2, t_3 : Time$

$\bullet\; stay_2(x, y, t_1, t_3) \Leftrightarrow t_1 \le t_2 \wedge t_2 \le t_3 \Rightarrow in(x, y, t_2)$ %(definition of stay$_2$)%

For the blends to be successful, the predicates *at*, *in*, $go_1$, $go_2$, $stay_1$ and $stay_2$ should be imported into the blend. The meanings of these predicates in the blend should support the interpretation of sentences like *Water boils <u>at</u> one hundred degrees*, *John <u>stayed</u> calm*, and *The inheritance <u>went</u> from John to Mary*, where spatial language is applied to non-spatial situations.

## 4.2 Results

The results shown in this section are also contained in the attached file `blends.casl`.

### 4.2.1 Experiment 1: Change in Place and Change in Temperature

In order to ensure that HDTP correctly generalizes the sort $Obj$ and the functions $place : Obj \to Fluent$ and $temp : Obj \to Temp$, it is necessary to add some 'dummy axioms' to the theory. The dummy axioms are needed because the implementation of HDTP used in this report skips symbols with no associated axioms. This is an issue with the implementation, not with HDTP itself.

The following axioms are added to CHANGE IN PLACE and CHANGE IN TEMPERATURE respectively:

$\forall\, x : Obj$
$\bullet\ x = x$ %(dummy axiom 1)%
$\bullet\ place(x) = place(x)$ %(dummy axiom 2)%

$\forall\, x : Obj$
$\bullet\ x = x$ %(dummy axiom 1)%
$\bullet\ temp(x) = temp(x)$ %(dummy axiom 2)%

When the dummy axioms are added, the COINVENT implementation of HDTP discovers the expected generic space, which is labeled GENERALIZATION0 (for the full structure of the generalization, see Appendix B.1). It also discovers the following substitutions generalizing CHANGE IN PLACE and CHANGE IN TEMPERATURE:

**view** MAPPING0_1 :
GENERALISATION0 **to** CHANGE_IN_PLACE =
*Obj* ↦ *Obj*, *Point_Temp* ↦ *Point*,
*Region_TempRegion* ↦ *Region*, *Time* ↦ *Time*,
*Fluent* ↦ *Fluent*, *betw* ↦ *betw*, *continuous* ↦ *continuous*,
*is_in* ↦ *is_in*, *G_G1841795* ↦ *place*, *precedes* ↦ *precedes*,
*precedes_neq* ↦ *precedes_neq*, *subset* ↦ *subset*,
*value_at* ↦ *value_at*
**end**

**view** MAPPING0_2 :
GENERALISATION0 **to** CHANGE_IN_TEMPERATURE =
*Obj* ↦ *Obj*, *Point_Temp* ↦ *Temp*,
*Region_TempRegion* ↦ *TempRegion*, *Time* ↦ *Time*,
*Fluent* ↦ *Fluent*, *betw* ↦ *betw*, *continuous* ↦ *continuous*,
*is_in* ↦ *is_in*, *G_G1841795* ↦ *temp*, *precedes* ↦ *precedes*,
*precedes_neq* ↦ *precedes_neq*, *subset* ↦ *subset*,

*value_at* $\mapsto$ *value_at*

**end**

As shown above, the system discovers that physical objects are analogous to physical objects, points to temperatures, spatial regions to temperature regions, place fluents to temperature fluents, and so on. The function $place : Obj \to Fluent$ in the CHANGE IN PLACE domain is correctly linked to $temp : Obj \to Temp$ in the CHANGE IN TEMPERATURE domain via the generalization $G\_G1841795 : Obj \to Fluent$.

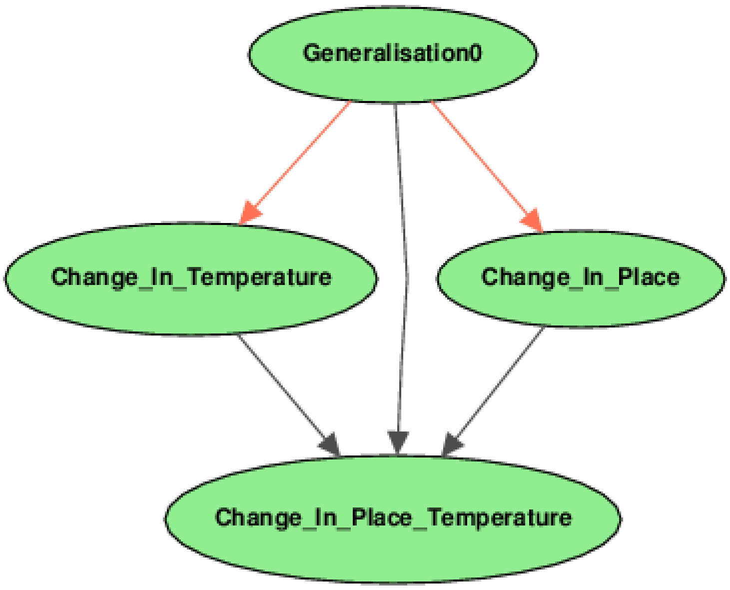


Figure 4.2: The blending diagram for CHANGE IN PLACE and CHANGE IN TEMPERATURE.

The system then computes the blend using the pushout of maps MAPPING0_1 and MAPPING0_2. The pushout is constructed automatically in Hets using the **combine** command:

**spec** CHANGE_IN_PLACE_TEMPERATURE =

**combine** *mapping0_1*, *mapping0_2*

**end**

The structure of the blend space CHANGE IN PLACE/TEMPERATURE is shown below. I have rewritten the theory to be more concise and readable, but it is the same up to a renaming of symbols.

**spec** CHANGE_IN_PLACE_TEMPERATURE =

**sorts** *Obj*, *Temp*, *Dist*, *TempRegion*, *Time*, *Fluent*

**preds** $\_\_ \leq \_\_ : Temp \times Temp$;

$\_\_<\_\_ : Dist \times Dist;$
$[\_\_\_\_\_\_] : Temp \times Temp \times Temp;$
$\_\_\in\_\_ : Temp \times TempRegion;$
$\_\_\subseteq\_\_ : TempRegion \times TempRegion;$
$\_\_\leq\_\_ : Time \times Time;$
$\_\_<\_\_ : Time \times Time;$
$continuous : Fluent;$
$at : Obj \times Temp \times Time;$
$in : Obj \times TempRegion \times Time;$
$go_1 : Obj \times Temp \times Temp \times Time \times Time;$
$go_2 : Obj \times TempRegion \times TempRegion \times Time \times Time;$
$stay_1 : Obj \times Temp \times Time \times Time;$
$stay_2 : Obj \times TempRegion \times Time \times Time$

**ops** $\_\_+\_\_ : Dist \times Dist \to Dist;$
$0 : Dist;$
$dist : Temp \times Temp \to Dist;$
$temp : Obj \to Fluent;$
$value\_at : Fluent \times Time \to Temp$

%temperature axioms%
$\forall\, x, y, z : Temp$
• $x \leq y \wedge y \leq x \Rightarrow y = x$ %(antisymmetry)%
• $x \leq y \wedge y \leq z \Rightarrow x \leq z$ %(transitivity)%
• $x \leq y \vee y \leq x$ %(totality)%

%distance axioms%
$\forall\, x, y, z : Dist$
• $x < y \Rightarrow \neg\, y < x$ %(antisymmetry)%
• $x < y \wedge y < z \Rightarrow x < z$ %(transitivity)%
• $x < y \vee y < x \vee x = y$ %(totality)%
• $0 < x \vee x = 0$ %(minimum element)%
• $x + (y + z) = (x + y) + z$ %(associativity)%
• $x + 0 = x$ %(identity element)%
• $x + y = y + x$ %(commutativity)%
• $x < y \Rightarrow x + z < y + z$ %(order preservation)%

%metric axioms%

$\forall\, x, y, z : Temp$

$\bullet\ dist(x, x) = 0$ %(indiscernability)%

$\bullet\ dist(x, y) = dist(y, x)$ %(symmetry)%

$\bullet\ dist(x, z) < dist(x, y) + dist(y, z)$

$\quad \vee\ dist(x, z) = dist(x, y) + dist(y, z)$ %(triangle inequality)%

%betweenness%

$\forall\, x, y, z, w : Temp$

$\bullet\ [x\ y\ z] \Leftrightarrow dist(x, z) = dist(x, y) + dist(y, z)$ %(definition of between 1)%

$\bullet\ [x\ y\ z] \Leftrightarrow (x \leq y \wedge y \leq z) \vee (z \leq y \wedge y \leq z)$ %(definition of between 2)%

$\bullet\ [x\ y\ z] \Rightarrow [z\ y\ x]$ %(symmetry)%

$\bullet\ [x\ y\ z] \wedge [y\ z\ x] \Rightarrow x = y \wedge y = z$ %(anticyclicity)%

$\bullet\ [x\ y\ z] \wedge [x\ z\ w] \Rightarrow [x\ y\ w]$ %(transitivity)%

$\bullet\ [x\ y\ z] \vee [y\ z\ x] \vee [z\ x\ y]$ %(totality)%

%region axioms%

$\forall\, x, y : TempRegion;\ z, v, w : Temp$

$\bullet\ x = y \Leftrightarrow (z \in x \Leftrightarrow z \in y)$ %(extensionality)%

$\bullet\ x \subseteq y \Leftrightarrow z \in x \Rightarrow z \in y$ %(definition of subset)%

%time axioms%

$\forall\, x, y, z : Time$

$\bullet\ x \leq y \wedge y \leq x \Rightarrow y = x$ %(antisymmetry)%

$\bullet\ x \leq y \wedge y \leq z \Rightarrow x \leq z$ %(transitivity)%

$\bullet\ x \leq y \vee y \leq x$ %(totality)%

$\bullet\ x < y \Leftrightarrow x \leq y \wedge \neg\, x = y$ %(definition of <)%

%fluent axioms%

$\forall\, x, y : Fluent;\ t : Time$

$\bullet\ x = y \Leftrightarrow value\_at(x, t) = value\_at(y, t)$ %(fluents are extensional)%

$\forall\, x : Obj \bullet continuous(temp(x))$ %(objects change continuously)%

%dummy axioms%

$\forall\, x : Obj$

$\bullet\ x = x$ %(dummy axiom 1)%

$\bullet\ temp(x) = temp(x)$ %(dummy axiom 2)%

%transferred concepts%

$\forall\, x : Obj;\, y, z : Temp;\, t_1, t_2, t_3 : Time$

- $at(x, y, t_1) \Leftrightarrow value\_at(temp(x), t_1) = y$ %(definition of at)%
- $go_1(x, y, z, t_1, t_2) \Leftrightarrow at(x, y, t_1) \wedge at(x, z, t_2) \wedge t_1 < t_2$ %(definition of go$_1$)%
- $stay_1(x, y, t_1, t_3) \Leftrightarrow t_1 \leq t_2 \wedge t_2 \leq t_3 \Rightarrow at(x, y, t_2)$ %(definition of stay$_1$)%

$\forall\, x : Obj;\, y, z : TempRegion;\, t_1, t_2, t_3 : Time$

- $in(x, y, t_1) \Leftrightarrow value\_at(temp(x), t_1) \in y$ %(definition of in)%
- $go_2(x, y, z, t_1, t_2) \Leftrightarrow in(x, y, t_1) \wedge in(x, z, t_2) \wedge t_1 < t_2$ %(definition of go$_2$)%
- $stay_2(x, y, t_1, t_3) \Leftrightarrow t_1 \leq t_2 \wedge t_2 \leq t_3 \Rightarrow in(x, y, t_2)$ %(definition of stay$_2$)%

**end**

As shown in the above theory, the blended theory contains temperatures, distances, temperature regions, times and fluents. Temperatures are totally ordered. Distances are totally ordered and have a concept of addition. The relationship between temperatures and distances is given by the metric axioms. The spatial concepts *at*, *in*, $go_1$, $go_2$, $stay_1$ and $stay_2$ have been imported into the blend.

### 4.2.2 Experiment 2: Change in Place and Change in Emotion

HDTP automatically discovers the expected generalization linking CHANGE IN PLACE and CHANGE IN EMOTION, which is labelled GENERALIZATION1 (for the full generalization, see Appendix B.2)[1]. The system also discovers the following substitutions linking CHANGE IN PLACE and CHANGE IN EMOTION

**view** MAPPING1_1 :

GENERALISATION1 **to** CHANGE_IN_PLACE =
*Obj_Person* ↦ *Obj*, *Point_Emot* ↦ *Point*,
*Region_EmotRegion* ↦ *Region*, *Time* ↦ *Time*,
*Fluent* ↦ *Fluent*, *betw* ↦ *betw*, *is_in* ↦ *is_in*,
*G_G5351027* ↦ *place*, *precedes* ↦ *precedes*,
*precedes_neq* ↦ *precedes_neq*, *subset* ↦ *subset*,
*value_at* ↦ *value_at*

**end**

**view** MAPPING1_2 :

[1] As before, this requires the addition of dummy axioms to physical space and emotion.

GENERALISATION1 **to** CHANGE_IN_EMOTION =
*Obj_Person* ↦ *Person*, *Point_Emot* ↦ *Emot*,
*Region_EmotRegion* ↦ *EmotRegion*, *Time* ↦ *Time*,
*Fluent* ↦ *Fluent*, *betw* ↦ *betw*, *is_in* ↦ *is_in*,
*G_G5351027* ↦ *emot*, *precedes* ↦ *precedes*,
*precedes_neq* ↦ *precedes_neq*, *subset* ↦ *subset*,
*value_at* ↦ *value_at*
**end**

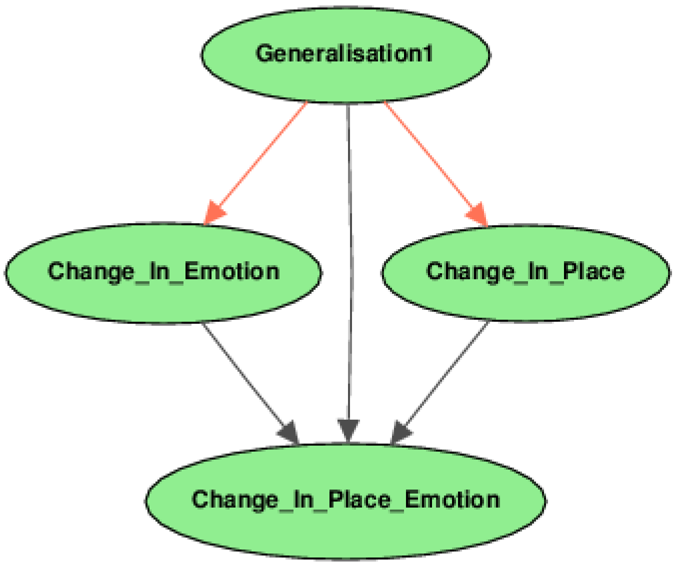


Figure 4.3: The blending diagram for CHANGE IN PLACE and CHANGE IN EMOTION.

As shown above, the system discovers that objects in the CHANGE IN PLACE domain are analogous to people in the CHANGE IN EMOTION domain, that points are analogous to emotions, and that spatial regions are analogous to emotion regions. The functions $place : Obj \to Fluent$ in the CHANGE IN PLACE domain and $emotion : Person \to Fluent$ in the CHANGE IN EMOTION domain are generalized via the function $G\_G5351027 : Obj\_Person \to Fluent$ in the generalization.

The structure of the blend space CHANGE IN PLACE/EMOTION is shown below. Again, the automatically generated theory has been rewritten to be more readable, but is the same up to a renaming of symbols.

**spec** CHANGE_IN_PLACE_EMOTION =

**sorts** *Person*, *Val*, *Arous*, *Emot*, *Dist*, *EmotRegion*, *Time*,
*Fluent*

**preds** $\_\_\leq\_\_ : Val \times Val$;
$[\_\_\_\_\_\_] : Val \times Val \times Val$;
$\_\_\leq\_\_ : Arous \times Arous$;
$[\_\_\_\_\_\_] : Arous \times Arous \times Arous$;
$[\_\_\_\_\_\_] : Emot \times Emot \times Emot$;
$\_\_<\_\_ : Dist \times Dist$;
$\_\_\in\_\_ : Emot \times EmotRegion$;
$\_\_\subseteq\_\_ : EmotRegion \times EmotRegion$;
$\_\_\leq\_\_ : Time \times Time$;
$\_\_<\_\_ : Time \times Time$;
$continuous : Fluent$;
$at : Person \times Emot \times Time$;
$in : Person \times EmotRegion \times Time$;
$go_1 : Person \times Emot \times Emot \times Time \times Time$;
$go_2 : Person \times EmotRegion \times EmotRegion \times Time \times Time$;
$stay_1 : Person \times Emot \times Time \times Time$;
$stay_2 : Person \times EmotRegion \times Time \times Time$

**ops** $min\_val : Val$;
$max\_val : Val$;
$min\_arous : Arous$;
$max\_arous : Arous$;
$val : Emot \rightarrow Val$;
$arous : Emot \rightarrow Arous$;
$\_\_+\_\_ : Dist \times Dist \rightarrow Dist$;
$0 : Dist$;
$dist : Emot \times Emot \rightarrow Dist$;
$emot : Person \rightarrow Fluent$;
$value\_at : Fluent \times Time \rightarrow Emot$

%valence axioms%

$\forall\, x, y, z : Val$

- $x \leq y \wedge y \leq x \Rightarrow y = x$ %(antisymmetry)%
- $x \leq y \wedge y \leq z \Rightarrow x \leq z$ %(transitivity)%
- $x \leq y \vee y \leq x$ %(totality)%
- $min\_val \leq x$ %(minimum element)%
- $x \leq max\_val$ %(maximum element)%
- $[x\ y\ z] \Leftrightarrow (x \leq y \wedge y \leq z) \vee (z \leq y \wedge y \leq z)$ %(definition of between)%

%arousal axioms%

$\forall\, x, y, z : Arous$

$\bullet\ x \leq y \wedge y \leq x \Rightarrow y = x$ %(antisymmetry)%

$\bullet\ x \leq y \wedge y \leq z \Rightarrow x \leq z$ %(transitivity)%

$\bullet\ x \leq y \vee y \leq x$ %(totality)%

$\bullet\ min\_arous \leq x$ %(minimum element)%

$\bullet\ x \leq max\_arous$ %(maximum element)%

$\bullet\ [x\ y\ z] \Leftrightarrow (x \leq y \wedge y \leq z) \vee (z \leq y \wedge y \leq z)$ %(definition of between)%

%emotion axioms%

$\forall\, x, y : Emot$

$\bullet\ x = y \Leftrightarrow val(x) = val(y) \wedge arous(x) = arous(y)$ %(equality)%

%distance axioms%

$\forall\, x, y, z : Dist$

$\bullet\ x < y \Rightarrow \neg\, y < x$ %(antisymmetry)%

$\bullet\ x < y \wedge y < z \Rightarrow x < z$ %(transitivity)%

$\bullet\ x < y \vee y < x \vee x = y$ %(totality)%

$\bullet\ 0 < x \vee x = 0$ %(minimum element)%

$\bullet\ x + (y + z) = (x + y) + z$ %(associativity)%

$\bullet\ x + 0 = x$ %(identity element)%

$\bullet\ x + y = y + x$ %(commutativity)%

$\bullet\ x < y \Rightarrow x + z < y + z$ %(order preservation)%

%metric axioms%

$\forall\, x, y, z : Emot$

$\bullet\ dist(x, x) = 0$ %(indiscernability)%

$\bullet\ dist(x, y) = dist(y, x)$ %(symmetry)%

$\bullet\ dist(x, z) < dist(x, y) + dist(y, z)$
$\quad \vee\ dist(x, z) = dist(x, y) + dist(y, z)$ %(triangle inequality)%

%betweenness for emotions%

$\forall\, x, y, z, w : Emot$

$\bullet\ [x\ y\ z] \Leftrightarrow dist(x, z) = dist(x, y) + dist(y, z)$ %(definition of between 1)%

$\bullet\ [x\ y\ z] \Leftrightarrow [val(x)\ val(y)\ val(z)]$
$\qquad \wedge\ [arous(x)\ arous(y)\ arous(z)]$ %(definition of between 2)%

- $[x\ y\ z] \Rightarrow [z\ y\ x]$ %(symmetry)%
- $[x\ y\ z] \wedge [y\ z\ x] \Rightarrow x = y \wedge y = z$ %(anticyclicity)%
- $[x\ y\ z] \wedge [x\ z\ w] \Rightarrow [x\ y\ w]$ %(transitivity)%

%region axioms%

$\forall\, x, y : EmotRegion;\ z, v, w : Emot$

- $x = y \Leftrightarrow (z \in x \Leftrightarrow z \in y)$ %(extensionality)%
- $x \subseteq y \Leftrightarrow z \in x \Rightarrow z \in y$ %(definition of subset)%

%time axioms%

$\forall\, x, y, z : Time$

- $x \leq y \wedge y \leq x \Rightarrow y = x$ %(antisymmetry)%
- $x \leq y \wedge y \leq z \Rightarrow x \leq z$ %(transitivity)%
- $x \leq y \vee y \leq x$ %(totality)%
- $x < y \Leftrightarrow x \leq y \wedge \neg\, x = y$ %(definition of <)%

%fluent axioms%

$\forall\, x, y : Fluent;\ t : Time$

- $x = y \Leftrightarrow value\_at(x, t) = value\_at(y, t)$ %(fluents are extensional)%

$\forall\, x : Person \bullet continuous(emot(x))$ %(change is continuous)%

%dummy axioms%

$\forall\, x : Person$

- $x = x$ %(dummy axiom 1)%
- $emot(x) = emot(x)$ %(dummy axiom 2)%

%transferred concepts%

$\forall\, x : Person;\ y, z : Emot;\ t_1, t_2, t_3 : Time$

- $at(x, y, t_1) \Leftrightarrow value\_at(emot(x), t_1) = y$ %(definition of at)%
- $go_1(x, y, z, t_1, t_2) \Leftrightarrow at(x, y, t_1) \wedge at(x, z, t_2) \wedge t_1 < t_2$ %(definition of $go_1$)%
- $stay_1(x, y, t_1, t_3) \Leftrightarrow t_1 \leq t_2 \wedge t_2 \leq t_3 \Rightarrow at(x, y, t_2)$ %(definition of $stay_1$)%

$\forall\, x : Person;\ y, z : EmotRegion;\ t_1, t_2, t_3 : Time$

- $in(x, y, t_1) \Leftrightarrow value\_at(emot(x), t_1) \in y$ %(definition of in)%
- $go_2(x, y, z, t_1, t_2) \Leftrightarrow in(x, y, t_1) \wedge in(x, z, t_2) \wedge t_1 < t_2$ %(definition of $go_2$)%
- $stay_2(x, y, t_1, t_3) \Leftrightarrow t_1 \leq t_2 \wedge t_2 \leq t_3 \Rightarrow in(x, y, t_2)$ %(definition of $stay_2$)%

**end**

As shown here, the blended theory contains valence values, arousal values, emotions, distances, emotion regions, times and fluents. Valence and arousal values are totally ordered and bounded. Emotions are the product of valence and arousal values. Distances are totally ordered and have an addition operation. The relationship between emotions and distances is governed by the metric axioms. The spatial concepts *at*, *in*, etc., are present in the blend.

### 4.2.3 Experiment 3: Change in Place and Inheritance

The system automatically discovers the expected generalization linking CHANGE IN PLACE and INHERITANCE, which is labelled GENERALIZATION2 (for the full generalization, see Appendix B.3). It discovers the following substitutions:

**view** MAPPING2_1 :

GENERALISATION2 **to** CHANGE_IN_PLACE =
*Obj_Heritable* $\mapsto$ *Obj*, *Point_Person* $\mapsto$ *Point*,
*Region_Set* $\mapsto$ *Region*, *Time* $\mapsto$ *Time*, *Fluent* $\mapsto$ *Fluent*,
*betw* $\mapsto$ *betw*, *continuous* $\mapsto$ *continuous*, *is_in* $\mapsto$ *is_in*,
*G_G5280638* $\mapsto$ *place*, *precedes* $\mapsto$ *precedes*,
*precedes_neq* $\mapsto$ *precedes_neq*, *subset* $\mapsto$ *subset*

**end**

**view** MAPPING2_2 :

GENERALISATION2 **to** INHERITANCE =
*Obj_Heritable* $\mapsto$ *Heritable*, *Point_Person* $\mapsto$ *Person*,
*Region_Set* $\mapsto$ *Set*, *Time* $\mapsto$ *Time*, *Fluent* $\mapsto$ *Fluent*,
*betw* $\mapsto$ *betw*, *continuous* $\mapsto$ *continuous*, *is_in* $\mapsto$ *is_in*,
*G_G5280638* $\mapsto$ *owner*, *precedes* $\mapsto$ *precedes*,
*precedes_neq* $\mapsto$ *precedes_neq*, *subset* $\mapsto$ *subset*

**end**

As shown above, HDTP discovers that objects in the CHANGE IN PLACE domain are analogous to heritable things in the INHERITANCE domain, that points are analogous to people, that spatial regions are analogous to sets of people, and so on. The functions $place : Obj \rightarrow Fluent$ and $owner : Heritable \rightarrow Fluent$ are generalized to $G\_G5280638 : Obj\_Heritable \rightarrow Fluent$ in the generalization.

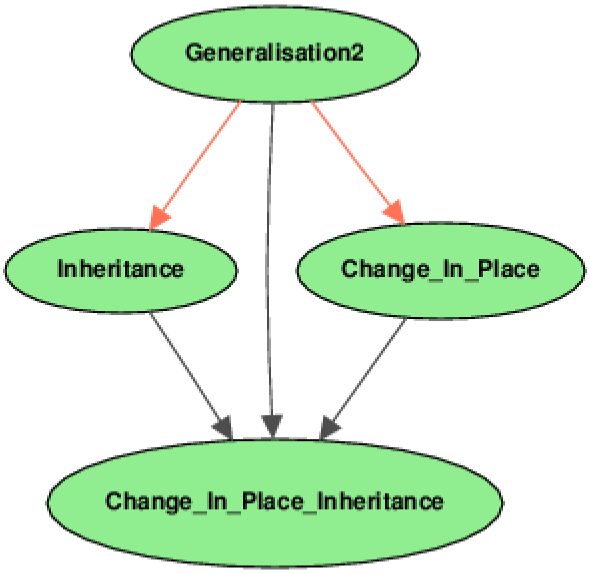


Figure 4.4: The blending diagram for CHANGE_IN_PLACE and INHERITANCE.

The blend space given by the colimit of maps MAPPING2_1, MAPPING2_2 is shown below. As before, the automatically generated theory has been rewritten to be more concise and readable.

**spec** CHANGE_IN_PLACE_INHERITANCE =

**sorts** *Person*, *Dist*, *Set*, *Heritable*, *Time*, *Fluent*
**preds** $ancestor : Person \times Person$;
$\_\_<\_\_ : Dist \times Dist$;
$[\_\_\_\_\_\_] : Person \times Person \times Person$;
$\_\_\in\_\_ : Person \times Set$;
$\_\_\subseteq\_\_ : Set \times Set$;
$\_\_\leq\_\_ : Time \times Time$;
$\_\_<\_\_ : Time \times Time$;
$continuous : Fluent$;
$at : Heritable \times Person \times Time$;
$in : Heritable \times Set \times Time$;
$go_1 : Heritable \times Person \times Person \times Time \times Time$;
$go_2 : Heritable \times Set \times Set \times Time \times Time$;
$stay_1 : Heritable \times Person \times Time \times Time$;
$stay_2 : Heritable \times Set \times Time \times Time$
**ops** $\_\_+\_\_ : Dist \times Dist \rightarrow Dist$;

$0 : Dist;$

$dist : Person \times Person \to Dist;$

$owner : Heritable \to Fluent;$

$value\_at : Fluent \times Time \to Person$

%ancestry axioms%

$\forall\, x, y, z : Person$

$\bullet\ ancestor(x, x)$ %(reflexivity)%

$\bullet\ ancestor(x, y) \wedge ancestor(y, x) \Rightarrow x = y$ %(antisymmetry)%

$\bullet\ ancestor(x, y) \wedge ancestor(y, z) \Rightarrow ancestor(x, z)$ %(transitivity)%

%distance axioms%

$\forall\, x, y, z : Dist$

$\bullet\ x < y \Rightarrow \neg\, y < x$ %(antisymmetry)%

$\bullet\ x < y \wedge y < z \Rightarrow x < z$ %(transitivity)%

$\bullet\ x < y \vee y < x \vee x = y$ %(totality)%

$\bullet\ 0 < x \vee x = 0$ %(minimum element)%

$\bullet\ x + (y + z) = (x + y) + z$ %(associativity)%

$\bullet\ x + 0 = x$ %(identity element)%

$\bullet\ x + y = y + x$ %(commutativity)%

$\bullet\ x < y \Rightarrow x + z < y + z$ %(order preservation)%

%metric axioms%

$\forall\, x, y, z : Emot$

$\bullet\ dist(x, x) = 0$ %(indiscernability)%

$\bullet\ dist(x, y) = dist(y, x)$ %(symmetry)%

$\bullet\ dist(x, z) < dist(x, y) + dist(y, z)$

$\quad \vee\ dist(x, z) = dist(x, y) + dist(y, z)$ %(triangle inequality)%

%betweenness%

$\forall\, x, y, z, w : Person$

$\bullet\ [x\ y\ z] \Leftrightarrow dist(x, z) = dist(x, y) + dist(y, z)$ %(definition of between 1)%

$\bullet\ [x\ y\ z] \Leftrightarrow (ancestor(x, y) \wedge ancestor(y, z))$

$\qquad \vee\ (ancestor(z, y) \wedge ancestor(y, x))$ %(definition of between 2)%

$\bullet\ [x\ y\ z] \Rightarrow [z\ y\ x]$ %(symmetry)%

$\bullet\ [x\ y\ z] \wedge [y\ z\ x] \Rightarrow x = y \wedge y = z$ %(anticyclicity)%

$\bullet\ [x\ y\ z] \wedge [x\ z\ w] \Rightarrow [x\ y\ w]$ %(transitivity)%

%region axioms%

$\forall\, x, y : \mathit{Set};\ z, v, w : \mathit{Person}$

$\bullet\ x = y \Leftrightarrow (z \in x \Leftrightarrow z \in y)$ %(extensionality)%

$\bullet\ x \subseteq y \Leftrightarrow z \in x \Rightarrow z \in y$ %(definition of subset)%

%time axioms%

$\forall\, x, y, z : \mathit{Time}$

$\bullet\ x \leq y \wedge y \leq x \Rightarrow y = x$ %(antisymmetry)%

$\bullet\ x \leq y \wedge y \leq z \Rightarrow x \leq z$ %(transitivity)%

$\bullet\ x \leq y \vee y \leq x$ %(totality)%

$\bullet\ x < y \Leftrightarrow x \leq y \wedge \neg\, x = y$ %(definition of <)%

%fluent axioms%

$\forall\, x, y : \mathit{Fluent};\ t : \mathit{Time}$

$\bullet\ x = y \Leftrightarrow \mathit{value\_at}(x, t) = \mathit{value\_at}(y, t)$ %(fluents are extensional)%

$\forall\, x : \mathit{Heritable}$

$\bullet\ \mathit{continuous}(\mathit{owner}(x))$ %(inheritance is continuous)%

$\forall\, x : \mathit{Fluent};\ t_1, t_2 : \mathit{Time}$

$\bullet\ t_1 < t_2 \Rightarrow \mathit{ancestor}(\mathit{value\_at}(x, t_1), \mathit{value\_at}(x, t_2))$ %(inheritance is downward)%

%transferred concepts%

$\forall\, x : \mathit{Heritable};\ y, z : \mathit{Person};\ t_1, t_2, t_3 : \mathit{Time}$

$\bullet\ \mathit{at}(x, y, t_1) \Leftrightarrow \mathit{value\_at}(\mathit{owner}(x), t_1) = y$ %(definition of at)%

$\bullet\ \mathit{go}_1(x, y, z, t_1, t_2) \Leftrightarrow \mathit{at}(x, y, t_1) \wedge \mathit{at}(x, z, t_2) \wedge t_1 < t_2$ %(definition of $\mathrm{go}_1$)%

$\bullet\ \mathit{stay}_1(x, y, t_1, t_3) \Leftrightarrow t_1 \leq t_2 \wedge t_2 \leq t_3 \Rightarrow \mathit{at}(x, y, t_2)$ %(definition of $\mathrm{stay}_1$)%

$\forall\, x : \mathit{Heritable};\ y, z : \mathit{Set};\ t_1, t_2, t_3 : \mathit{Time}$

$\bullet\ \mathit{in}(x, y, t_1) \Leftrightarrow \mathit{value\_at}(\mathit{owner}(x), t_1) \in y$ %(definition of in)%

$\bullet\ \mathit{go}_2(x, y, z, t_1, t_2) \Leftrightarrow \mathit{in}(x, y, t_1) \wedge \mathit{in}(x, z, t_2) \wedge t_1 < t_2$ %(definition of $\mathrm{go}_2$)%

$\bullet\ \mathit{stay}_2(x, y, t_1, t_3) \Leftrightarrow t_1 \leq t_2 \wedge t_2 \leq t_3 \Rightarrow \mathit{in}(x, y, t_2)$ %(definition of $\mathrm{stay}_2$)%

**end**

The blended theory CHANGE IN PLACE/INHERITANCE contains people, distances, sets of people, heritable objects, times and fluents. People are partially ordered by the ancestor relation. Distances are totally ordered and have an addition operation. The relationship between people and distances is governed by the metric axioms. The blended theory also contains the spatial concepts *at*, *in*, $go_1$, $go_2$, $stay_1$ and $stay_2$.

## 4.3 Evaluation

This section evaluates the results according to the Evaluation Criteria given in Section 4.1.

### 4.3.1 Target generic space

The three generalizations, GENERALIZATION0, GENERALIZATION1 and GENERALIZATION2 (see Appendices B.1, B.2 and B.3), which link CHANGE IN PLACE with CHANGE IN TEMPERATURE, CHANGE IN EMOTION and INHERITANCE respectively, each contain the target generic space CHANGE IN CONCEPTUAL SPACE. This can be seen by renaming symbols as follows:

GENERALIZATION0 $Point\_Temp \mapsto State$; $Region\_TempRegion \mapsto Set$; $betw \mapsto [\_\_\_\_\_\_]$; $is\_in \mapsto \_\_ \in \_\_$; $G\_G40288716 \rightarrow state$; $precedes \mapsto \_\_ \leq \_\_$; $precedes\_neq \mapsto \_\_ < \_\_$; $subset \mapsto \_\_ \subseteq \_\_$.

GENERALIZATION1 $Obj\_Person \mapsto Obj$; $Point\_Emot \mapsto State$; $Region\_EmotRegion \mapsto Set$; $betw \mapsto [\_\_\_\_\_\_]$; $is\_in \mapsto \_\_ \in \_\_$; $G\_G5351027 \mapsto state$; $precedes \mapsto \_\_ \leq \_\_$; $precedes\_neq \mapsto \_\_ < \_\_$; $subset \mapsto \_\_ \subseteq \_\_$.

GENERALIZATION2 $Obj\_Heritable \mapsto Obj$; $Point\_Person \mapsto State$; $Region\_Set \mapsto Set$; $betw \mapsto [\_\_\_\_\_\_]$; $is\_in \mapsto \_\_ \in \_\_$; $G\_G5280638 \mapsto state$; $precedes \mapsto \_\_ \leq \_\_$; $precedes\_neq \mapsto \_\_ < \_\_$; $subset \mapsto \_\_ \subseteq \_\_$.

The three generalizations contain all the axioms in CHANGE IN CONCEPTUAL SPACE, including the betweenness axioms, the region axioms, the time axioms, and the axiom of extensionality for fluents. In addition to the subtheory CHANGE IN CONCEPTUAL SPACE, the three generalizations also contain the two dummy axioms which are added to ensure that $Obj$ and $state$ are appropriately generalized.

In the case of GENERALIZATION0 and GENERALIZATION2, there is also an axiom which ensures that fluents change continuously. This happens because CHANGE IN PLACE shares with both CHANGE IN TEMPERATURE and INHERITANCE the requirement that fluents change continuously. The CHANGE IN EMOTION domain does not contain this axiom, so it is not present in GENERALIZATION1.

The fact that all three generalizations contain the target CHANGE IN CONCEPTUAL SPACE supports the Research Hypothesis 3, that analogies between the spatial domain and other domains include the conceptual space structure. As we shall see in the next section, the presence of betweenness in the generalization ensures an interaction between the metric structure of physical space and the order structure in the other domains.

### 4.3.2 Consistency of the blends

The three blends, CHANGE IN PLACE/TEMPERATURE, CHANGE IN PLACE/EMOTION and CHANGE IN PLACE/INHERITANCE, all contain two alternative definitions of betweenness. The first definition is $[x\ y\ z] \Leftrightarrow dist(x,z) = dist(x,y) + dist(y,z)$, which originates in the theory of physical space. The second definition originates in the other input space. In order to be consistent, there must be an interpretation of betweenness in the blend which satisfies both definitions.

For CHANGE IN PLACE/TEMPERATURE, the second definition of betweenness is $[x\ y\ z] \Leftrightarrow (x \leq y \leq z) \vee (z \leq y \leq x)$. For the theory to be consistent, we must have that $(x \leq y \leq z) \vee (z \leq y \leq x) \Leftrightarrow dist(x,z) = dist(x,y) + dist(y,z)$, by the transitivity of the biconditional. This is a satisfiable requirement. For example, it is satisfied by the usual metric for the real numbers $dist(x,y) = |x - y|$. Therefore, the two definitions of distance are consistent.

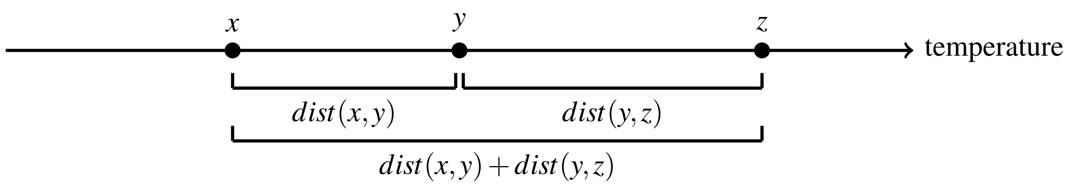


Figure 4.5: Compatibility of betweenness requires that for temperatures $x \leq y \leq z$, the distance $xz$ is the distance $xy$ plus the distance $yz$.

The requirement that $(x \leq y \leq z) \vee (z \leq y \leq x) \Leftrightarrow dist(x,z) = dist(x,y) + dist(y,z)$ has the desirable effect of enforcing a compatibility between the metric structure of physical space and the order structure of temperatures, in the sense that the topology induced by the metric in CHANGE IN PLACE/TEMPERATURE is the same as the topology induced by the total order[2].

In CHANGE IN PLACE/EMOTION, the second definition of betweenness is

$$[x\ y\ z] \Leftrightarrow [val(x)\ val(y)\ val(z)] \wedge [arous(x)\ arous(y)\ arous(z)]$$

The two definitions require that

$$[val(x)\ val(y)\ val(z)] \wedge [arous(x)\ arous(y)\ arous(z)] \Leftrightarrow dist(x,z) = dist(x,y) + dist(y,z)$$

This condition is satisfied by the *rectilinear metric* (sometimes called the *Manhattan metric*)

[2]To see this, consider that the open sets in the metric topology are open balls of the form $B(x,r) = \{p|dist(x,p) < r\}$, and the open sets in the order topology are open intervals of the form $(x,y) = \{p|x < p < y\}$. But every open ball $B(x,r)$ is an open interval of the form $(x+r, x-r)$ so the two topologies have the same basis of open sets are are therefore identical.

where the distance between two points is given by the sum of their separations along each axis (see Figure 4.6).

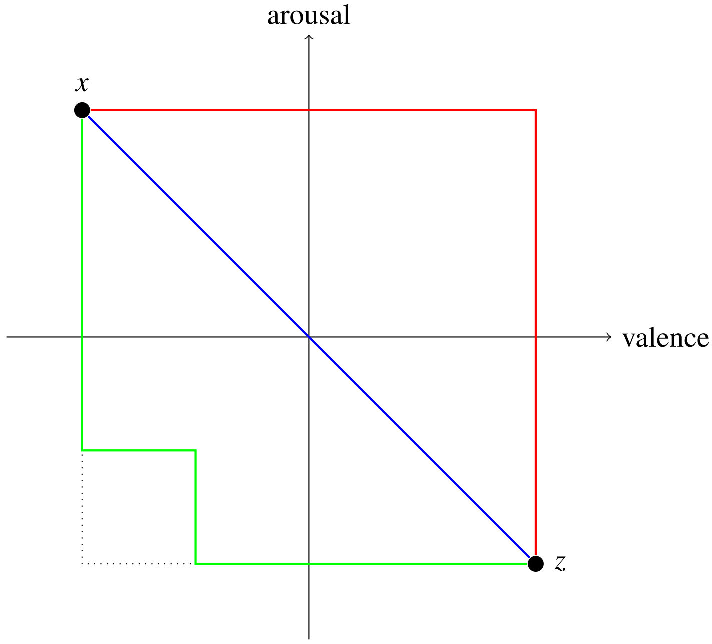


Figure 4.6: With a rectilinear metric, there is no unique shortest path from $x$ to $z$; the red, green and blue paths all have the same length.

Unlike in CHANGE IN PLACE/TEMPERATURE, it is not straightforward to show that the topology induced by the metric and the order are the same in CHANGE IN PLACE/EMOTION. However, whereas the two structures may not agree on open sets, they certainly agree on convex sets, since a convex set is defined using the betweenness relation.

Finally, in the case of CHANGE IN PLACE/INHERITANCE, the second definition of betweenness is given by $[x\ y\ z] \Leftrightarrow (ancestor(x,y) \wedge ancestor(y,z)) \vee (ancestor(z,y) \wedge ancestor(y,x))$. As before, the two definitions must be consistent. The natural interpretation of distance in the INHERITANCE domain is to define the distance between two people to be the number of parent edges separating them. For example, in Figure 4.7, the distance between Harry and Jane is 1 because Harry is Jane's father, whereas the distance between John and Jane is 2 because John is Jane's grandfather.

However, to ensure consistency, we cannot allow distances between people who are not ordered with respect to each other. For example, the distance between Harry and Jane is 1, and the distance between Harry and Ben is 1, but we cannot allow the distance between the siblings Jane and Ben to be 2, because this violates the condition that $(ancestor(x,y) \wedge ancestor(y,z)) \vee (ancestor(z,y) \wedge ancestor(y,x)) \Leftrightarrow dist(x,z) = dist(x,y) + dist(y,z)$.

There are two ways to alter the metric so as to meet this condition. One is to say that the distance is undefined if the two people are not ordered with respect to each other. This is not

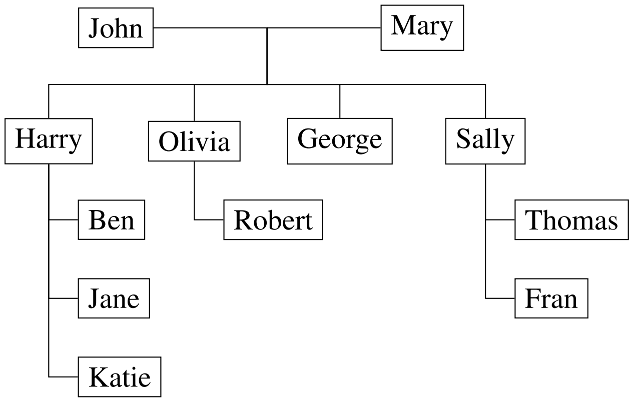


Figure 4.7: A family tree.

ideal because functions in first-order theories are usually assumed to be total, and the function *dist* is total in the input space PHYSICAL_SPACE. The other solution is to introduce an infinite distance, and define the value of the *dist* function to be infinite whenever the two inputs are respectively unordered. Additional axioms would be required to ensure that (a) the infinite distance is larger than all other distances, and (b) the infinite distance cannot be reached by taking finite sums of other distances. If this solution is adopted, then the metric and the partial order agree on convex sets.

In summary, all three blends CHANGE IN PLACE/TEMPERATURE, CHANGE IN PLACE/EMOTION and CHANGE IN PLACE/INHERITANCE are consistent, although figuring out interpretations which make them consistent takes some work. This finding supports Research Hypothesis 1, that physical space can be consistently blended with other domains.

### 4.3.3 Transferred spatial concepts

Each of the blends contains the predicates *at*, *in*, $go_1$, $go_2$, $stay_1$ and $stay_2$, which originate in the CHANGE IN PLACE domain. Due to this transfer of information, the blend spaces can be used to represent sentences which could not be represented in the original theories CHANGE IN TEMPERATURE, CHANGE IN EMOTION and INHERITANCE.

For example, consider the following sentences containing *in* and *at*:

1. *Water boils at one hundred degrees.*

2. *John's in a depression.*

3. *The heirloom is in Mary's line.*

The meanings of these sentences could not be represented in the original CHANGE IN TEMPERATURE, CHANGE IN EMOTION and INHERITANCE domains, because the meaning of the words *at* and *in* was only defined in the CHANGE IN PLACE domain. However, once the blend has been constructed, they can be assigned representations like the following (the individual *present* corresponds to the time of utterance):

1. $\forall x : Obj\ .\ water(x) \wedge \underline{at}(x, 100°C) \Rightarrow boil(x)$

2. $\underline{in}(John, depression, present)$

3. $\underline{in}(heirloom, line(Mary), present)$

This use of a spatial preposition like *at* or *in* to refer to 'location' in a property space like temperature or emotion is not specific to English. For example, in Chinese, the word *zhi-zhong*, 'inside', which is used to talk about location in a region, can also be used to talk about having an emotion [Yu, 1998].

| | | | | | | |
|---|---|---|---|---|---|---|
| *ta* | *taozui* | *zai* | *zhe* | *jude* | *xiyue* | *zhi-zhong* |
| he | be-intoxicated | PRT | this | huge | joy | inside |

"He is intoxicated in a huge joy."

In addition to the concept of location, the concept of motion is also meaningful in the blended theories. Consider the following sentences:

1. The food went from hot to cold.

2. John went from happy to angry.

3. The inheritance went from John to Katie.

As before, the meaning of these sentences could not be represented in the original CHANGE IN TEMPERATURE, CHANGE IN EMOTION and INHERITANCE domains because the meaning of the *X went from Y to Z* construction was only defined in the CHANGE IN PLACE domain. However, having constructed the blends, they can now be assigned representations like the following:

1. $\exists t_1, t_2 : Time\ .\ \underline{go_2}(food, hot, cold, t_1, t_2) \wedge t_1 < present\ \wedge\ t_2 < present$

2. $\exists t_1, t_2 : Time\ .\ \underline{go_2}(John, happy, angry, t_1, t_2)\ \wedge\ t_1 < present\ \wedge\ t_2 < present$

3. $\exists t_1, t_2 : Time\ .\ \underline{go_1}(inheritance, John, Katie, t_1, t_2) \wedge t_1 < present \wedge t_2 < present$

As shown above, sentences 1 and 2 are examples of $go_2$ because they describe motion between temperature regions and emotion regions respectively. Sentence 3 is an example of $go_1$ because it describes motion between exact locations, (the exact locations in this case being individual owners). The axiom of continuity ensures that the motion described in sentences


Figure 4.8: *went from hot to cold* in the temperature domain.

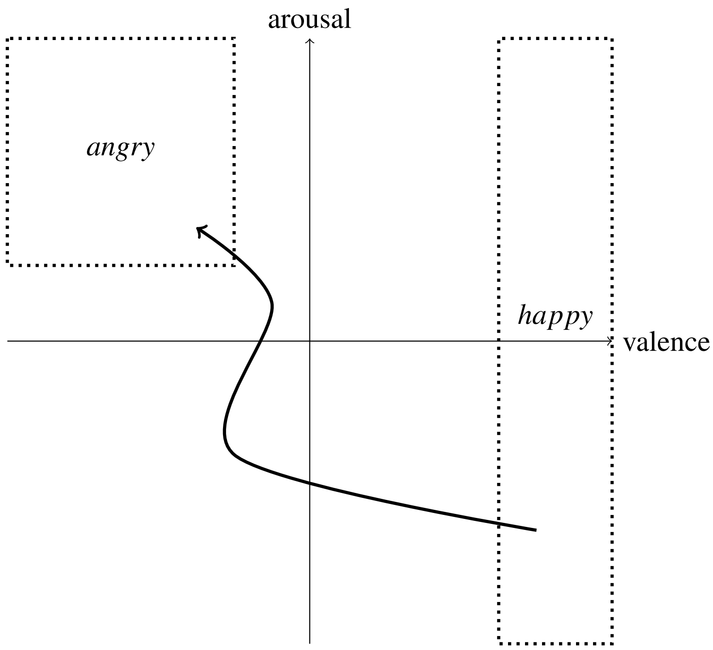


Figure 4.9: *went from happy to angry* in the emotion domain.

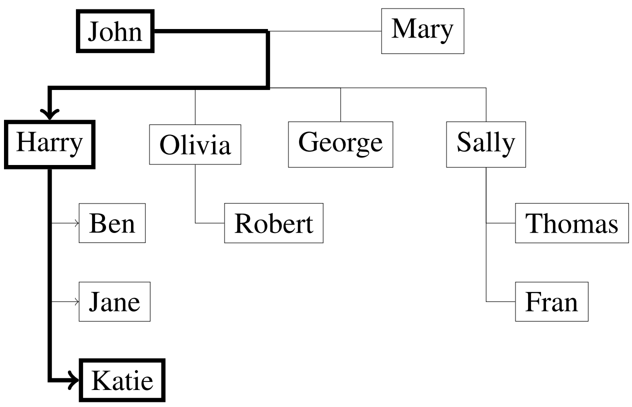


Figure 4.10: *went from John to Katie* in the ancestry domain.

1-3 respects the topology of the domain. See Figures 4.8, 4.9 and 4.10 for illustrations of these three sentences.

Sentences involving the word *stay* can also be interpreted in the blend theories. Consider the following sentences:

1. *The water stayed at thirty degrees.*
2. *John stayed angry for three hours.*
3. *The heirloom stayed in Mary's line.*

As before, these sentences could not be represented in the original theories CHANGE IN TEMPERATURE, CHANGE IN EMOTION and INHERITANCE. However, once the blends have been constructed, they can be assigned representations like the following:

1. $\exists t_1,t_2 : Time\ .\ \underline{stay_1}(water, 30°C, t_1, t_2)\ \wedge\ t_1 < present\ \wedge\ t_2 < present$
2. $\exists t_1,t_2 : Time\ .\ \underline{stay_2}(John, angry, t_1, t_2)\ \wedge\ t_1 < present\ \wedge\ t_2 < present \wedge t_2 - t_1 = 3hrs$
3. $\exists t_1,t_2 : Time\ .\ \underline{stay_2}(heirloom, line(Mary), t_1, t_2) \wedge t_1 < present \wedge t_2 < present$

Here, sentence 1 is an example of $stay_1$, because it describes an object staying at $30°C$ degrees, which is a precise location in temperature space. By contrast, sentences 2 and 3 are examples of $stay_2$ because they describe an object remaining in an emotion region and an ancestry region (lineage) respectively.

Not all the concepts which are available in the blend are lexically realised. For example, in English the concept *in* is not used in the temperature domain - one does not say **The temperature was in cold*. However, there are languages where such sentences are more common. For example, in Latin one can say *in frigore maneo*, 'I remain in coldness' [Fedriani, 2011].

To give another example, English does not use the word *at* to express possession although this possibility is made available by the blend CHANGE IN PLACE/INHERITANCE. Other languages do make use of this concept. For example, Estonian (a), Irish (b) and Lezgian (c) all use a locative expression analogous to 'at' to encode possession.

(a) Estonian [Payne, 1997]

| | | |
|---|---|---|
| *lapsel* | *on* | *piima* |
| child:LOC | be | milk |

"The child has milk."
(literally: "Milk is at the child.")

(b) Irish [Stassen, 2009]

| | | |
|---|---|---|
| *ta* | *airgead* | *aig-e* |
| be | money | at-3SG |

"He has money."
(literally: "Money is at him.")

(c) Lezgian [Stassen, 2009]

| | | |
|---|---|---|
| *dusmanriw* | *tup-ar* | *gwa-c* |
| enemy | cannon-PL | be.at-NEG |

"The enemy does not have cannons"
(literally: "There are no cannons at the enemy.")

These examples show that there is a great deal of language-specificity in which concepts get linguistically encoded. Nevertheless, English speakers still have access to a mental space in which people are identified with points, since this conceptualization is crucial to sentences like *The money went from John to Mary*.

The successful transfer of the concepts *at*, *in*, $go_1$, $go_2$, $stay_1$ and $stay_2$ into the CHANGE IN TEMPERATURE, CHANGE IN EMOTION and INHERITANCE domains supports Research Hypothesis 2, that conceptual blending can model the transfer of spatial concepts into new domains.

## 4.4 Experiment 4: Change in Temperature and Change in Emotion

The theories CHANGE IN TEMPERATURE, CHANGE IN EMOTION and INHERITANCE can be blended with each other in addition to CHANGE IN PLACE. However, doing so does not always generate interesting blends. One potentially interesting possibility would be a combination of CHANGE IN TEMPERATURE and CHANGE IN EMOTION in which temperatures are identified with arousal values. This blend might help explain sentences like *He was hot with rage* and *She felt as cool as a cucumber*, in which high arousal is associated with heat and low arousal is associated with coldness.

As before, I assume that this blend is linguistically guided. Suppose a language learner who knows the meaning of *argument* and *heat* encounters the sentence *The argument became heated*. This sentence is anomalous because arguments are not normally the kind of entities which can have temperatures. The sentence therefore prompts the learner to combine her understanding of change in temperature and change in emotion. This combination can be modelled using conceptual blending.

The generalization corresponding to this blend of CHANGE IN TEMPERATURE and CHANGE IN EMOTION is discovered automatically by HDTP and labelled GENERALIZATION3 (see Appendix B.4 for the full structure of the generalization). The substitutions are shown below:

**view** MAPPING3_1 :

GENERALISATION3 **to** CHANGE_IN_TEMPERATURE =
*Obj_Person* $\mapsto$ *Obj*, *Fluent* $\mapsto$ *Fluent*, *Time* $\mapsto$ *Time*,
*TempRegion_EmotRegion* $\mapsto$ *TempRegion*,
*Temp_Emot* $\mapsto$ *Temp*, *Temp_Arous* $\mapsto$ *Temp*, *betw* $\mapsto$ *betw*,
*G_G9476510* $\mapsto$ *colder*, *is_in* $\mapsto$ *is_in*, *precedes* $\mapsto$ *precedes*,
*precedes_neq* $\mapsto$ *precedes_neq*, *subset* $\mapsto$ *subset*,
*G_G9483970* $\mapsto$ *temp*, *value_at* $\mapsto$ *value_at*

**end**

**view** MAPPING3_2 :

GENERALISATION3 **to** CHANGE_IN_EMOTION =
*Obj_Person* $\mapsto$ *Person*, *Fluent* $\mapsto$ *Fluent*, *Time* $\mapsto$ *Time*,
*TempRegion_EmotRegion* $\mapsto$ *EmotRegion*,
*Temp_Emot* $\mapsto$ *Emot*, *Temp_Arous* $\mapsto$ *Arous*, *betw* $\mapsto$ *betw*,
*G_G9476510* $\mapsto$ *calmer*, *is_in* $\mapsto$ *is_in*, *precedes* $\mapsto$ *precedes*,
*precedes_neq* $\mapsto$ *precedes_neq*, *subset* $\mapsto$ *subset*,
*G_G9483970* $\mapsto$ *emot*, *value_at* $\mapsto$ *value_at*

**end**

As shown above, HDTP discovers that temperatures have some structure in common with arousal values, which is captured by the sort *Temp_Arous*. However, temperatures also have structure in common with emotions (such as betweenness, membership in regions, being the value of fluents, etc.), which is captured by the sort *Temp_Emot*. In the CHANGE IN EMOTION domain, *Temp_Arous* covers arousal values and *Temp_Emot* covers emotion values, whereas in the CHANGE IN TEMPERATURE domain *Temp_Arous* and *Temp_Emot* both cover temperatures.

This double cover of the *Temp* sort is problematic because it means that emotions and arousal values must be conflated in the blend in order to satisfy the pushout property. This is not desirable because emotions were originally the product sort of arousal and valence, and the blended theory eliminates this product structure.

One solution to this problem is to weaken the theory of CHANGE IN TEMPERATURE prior to blending, so that it only shares structure with either emotions or arousal values (see Figure 4.11). If we choose arousal values, then the resulting theory is one in which increased arousal is identified with increased temperature, allowing us to represent sentences like *He became heated*. For more on weakening input theories, see Section 5.3.

The 'double cover' issue also arises when blending CHANGE IN EMOTION and INHERITANCE. People in the INHERITANCE domain have some structure in common with valence/arousal

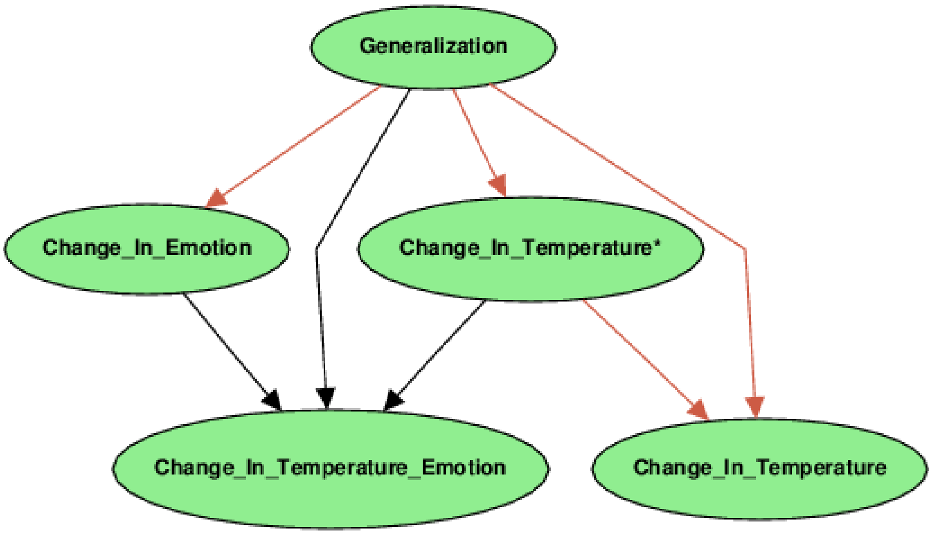


Figure 4.11: For CHANGE IN TEMPERATURE to be combined with CHANGE IN EMOTION in such a way that the structure of CHANGE IN EMOTION is respected, it must first be weakened to CHANGE IN TEMPERATURE*.

values (the partial order axioms), but also some structure in common with emotions (betweenness, belonging to regions, etc.). This results in the double cover of *Person* by both *Person_Arous* and *Person_Emot*, which again eliminates the product structure of the CHANGE IN EMOTION domain by collapsing arousal values and emotions.

The presence of a many-to-one substitution from the generalization to one of the input spaces always results in the collapse of structure, suggesting that it should be disfavoured by the cost function.

## 4.5 Summary

This chapter has presented four experiments. Experiments 1, 2 and 3 test the Research Hypotheses by blending the theory of CHANGE IN PLACE with CHANGE IN TEMPERATURE, CHANGE IN EMOTION and INHERITENCE respectively. These experiments were found to meet the Evaluation Criteria. Experiment 4 blended CHANGE IN PLACE and CHANGE IN TEMPERATURE for the sake of completeness, but found problems resulting from a many-to-one substitution map.

# Chapter 5

# Conclusion

## 5.1 Confirmation of Research Hypotheses

Research Hypothesis 1 was that computational cognitive blending could be used to discover consistent blends between physical space and other domains. Experiments 1, 2 and 3 are evidence towards this hypothesis because each results in a mathematically consistent combination of CHANGE IN PLACE with another domain. The results suggest that it should be possible to blend CHANGE IN PLACE with other theories of change, including CHANGE IN WEIGHT, CHANGE IN NUMBER, CHANGE IN COLOUR, etc.

Research Hypothesis 2 was that that conceptual blending could model the transfer of spatial concepts into other domains. Experiments 1, 2 and 3 count as evidence for this hypothesis, since in each case the predicates *at*, *in*, $go_1$, $go_2$, $stay_1$, and $stay_2$ were transferred correctly from CHANGE IN PLACE into a new domain. This suggests that other spatial concepts, such as *near*/*close*, *through*, *via*, and *on* could also be transferred by the same mechanism.

The final Research Hypothesis was that the geometric structure of a conceptual space can support analogies between physical space and other domains. This is supported by Experiments 1, 2 and 3 because in each case the generic space contained the theory CHANGE IN CONCEPTUAL SPACE, which contains the geometric structure of a conceptual space proposed by Gärdenfors. The betweenness relation acted as a conceptual 'glue' which resulted in an interaction between the metric structure of physical space and the order structure of temperatures, emotions, and lines of descent.

## 5.2 Theoretical Contributions

Previous accounts of spatial transfer, such as Conceptual Metaphor theory and Jackendoff's Conceptual Structure, have left open the question of exactly how spatial concepts are transferred into new domains during language acquisition. Both frameworks fall back on terms like 'analogy' or 'metaphor' with little explanation of the underlying mechanism. The main contribution of this report is that it offers a proof-of-concept account of how semantic transfer might happen, based on a mathematically sound and computationally-feasible framework. Computational conceptual blending can be seen as modelling the cognitive process which takes place when a learner encounters a sentence like *The food went from hot to cold* for the first time.

The conceptual blending account highlights the importance of a logical, axiomatic approach to semantic transfer. Frameworks like Conceptual Metaphor theory and Conceptual Structure are not based on logic, and as a result it is difficult to see how they could be applied to reasoning outside of the natural language context in which they have been proposed. By contrast, domains like CHANGE IN PLACE and CHANGE IN TEMPERATURE are needed not only for representing the meanings of words like *go* or *hot*, but also for interacting with and reasoning about one's environment. A blend like CHANGE IN PLACE/TEMPERATURE is not a closed book, but rather an open-ended collection of knowledge which can be added to as new theorems are discovered.

Finally, the conceptual blending account also highlights the importance of representing the differences between domains. Domains like temperature and emotion should not be lumped together into a single domain of 'properties', as in Conceptual Metaphor theory and Conceptual Structure, because they have different mathematical structures, which concepts like *in*, *go*, *stay*, etc., must respect. For example, continuous change means something different in the CHANGE IN TEMPERATURE, CHANGE IN EMOTION and ANCESTRY domains because the topology of temperatures is given by a total order, the topology of ancestry is given by a partial order, and the topology of emotions is the product topology of arousal and valence.

## 5.3 Future Research

The theory of CHANGE IN PLACE used in this report was abstract enough to be easily blended with other domains like CHANGE IN TEMPERATURE. However, if more detail was added to the theory, then elements of it would be incompatible with other domains. For example, if CHANGE IN PLACE included the axiom that two objects may not have the same location at the same time, then this would be incompatible with the CHANGE IN TEMPERATURE domain, because two objects may have the same temperature at the same time.

More work needs to be done on weakening the input theories appropriately, so that only compatible information appears in the blend. Weakening can be driven by the discovery of an inconsistency in the blended theory, or by the reduction the cost function associated with the generalization. For example, if CHANGE IN TEMPERATURE contains an instance of two objects having the same temperature at the same time, then this will result in an inconsistency unless CHANGE IN PLACE or CHANGE IN TEMPERATURE is appropriately weakened. One technique which has been proposed for weakening the input theories is *Amalgamation* (see Bou et al. [2014] for an example). A more complete analysis of spatial transfer would show how a technique like Amalgamation can be used to weaken the input theories prior to blending.

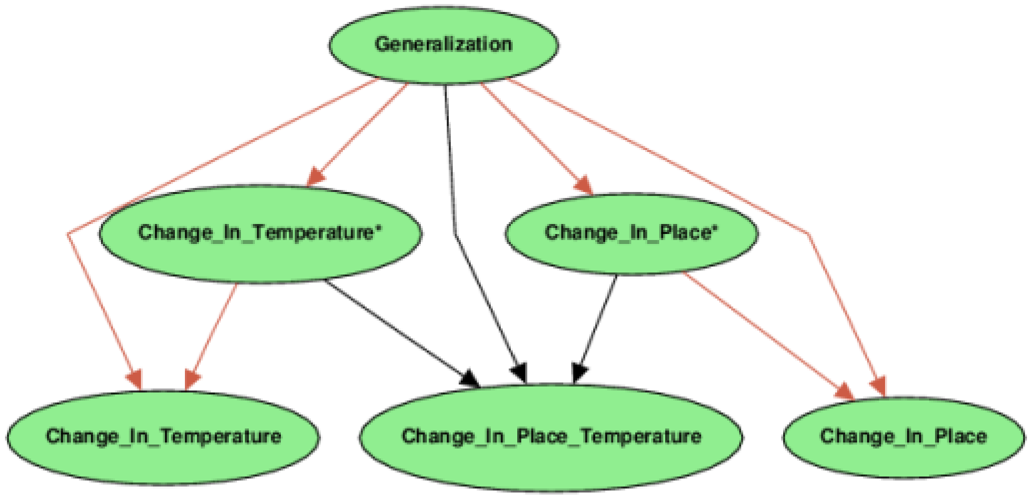


Figure 5.1: In a more complete account of spatial transfer, the theories CHANGE IN PLACE and CHANGE IN TEMPERATURE would be weakened to CHANGE IN PLACE* and CHANGE IN TEMPERATURE* prior to blending, to ensure that the blend space is consistent or to reduce the cost of the generalization.

A more detailed account of spatial transfer would also show how CHANGE IN PLACE can be blended with domains other than temperature, emotion and inheritance. Some domains which can support spatial concepts, such as the colour domain (e.g. *The sky went from blue to red*) or the number domain (e.g. *The number of people in the room went from 5 to 10*), have a fairly well-understood geometric structure. Other domains, such as information (e.g. *The news went from town to town*), or taste (e.g. *The drink stayed bitter*) have not been as extensively studied. Developing theories of conceptual domains will improve not only our understanding of spatial transfer, but also our general understanding of human reasoning and natural language semantics.

# Appendix A

## A.1 Change in Place

**spec** CHANGE_IN_PLACE $=$

**sort** *Obj*

**sort** *Point*

**sort** *Dist*

**sort** *Region*

**sort** *Time*

**sort** *Fluent*

**pred** $smaller : Dist \times Dist$

**pred** $betw : Point \times Point \times Point$

**pred** $is\_in : Point \times Region$

**pred** $subset : Region \times Region$

**pred** $precedes : Time \times Time$

**pred** $precedes\_neq : Time \times Time$

**pred** $continuous : Fluent$

**pred** $at : Obj \times Point \times Time$

**pred** $in : Obj \times Region \times Time$

**pred** $go\_1 : Obj \times Point \times Point \times Time \times Time$

**pred** $go\_2 : Obj \times Region \times Region \times Time \times Time$

**pred** $stay\_1 : Obj \times Point \times Time \times Time$

**pred** $stay\_2 : Obj \times Region \times Time \times Time$

**op** $plus : Dist \times Dist \to Dist$

**op** $0 : Dist$

**op** $dist : Point \times Point \to Dist$

**op** $place : Obj \to Fluent$

**op** $value\_at : Fluent \times Time \to Point$

$\bullet\ \forall\, x, y : Dist \bullet smaller(x, y) \Rightarrow \neg\, smaller(y, x)$

- $\forall\, x, y, z : Dist$
  $\bullet\ smaller(x, y) \land smaller(y, z) \Rightarrow smaller(x, z)$
- $\forall\, x, y : Dist \bullet smaller(x, y) \lor smaller(y, x) \lor x = y$
- $\forall\, x : Dist \bullet smaller(0, x) \lor x = 0$
- $\forall\, x, y, z : Dist \bullet plus(x, plus(y, z)) = plus(plus(x, y), z)$
- $\forall\, x : Dist \bullet plus(x, 0) = x$
- $\forall\, x, y : Dist \bullet plus(x, y) = plus(y, x)$
- $\forall\, x, y, z : Dist$
  $\bullet\ smaller(x, y) \Rightarrow smaller(plus(x, z), plus(y, z))$
- $\forall\, x : Point \bullet dist(x, x) = 0$
- $\forall\, x, y : Point \bullet dist(x, y) = dist(y, x)$
- $\forall\, x, y, z : Point$
  $\bullet\ smaller(dist(x, z), plus(dist(x, y), dist(y, z)))$
  $\lor\ dist(x, z) = plus(dist(x, y), dist(y, z))$
- $\forall\, x, y, z : Point$
  $\bullet\ betw(y, x, z) \Leftrightarrow dist(x, z) = plus(dist(x, y), dist(y, z))$
- $\forall\, x, y, z : Point \bullet betw(y, x, z) \Rightarrow betw(y, z, x)$
- $\forall\, x, y, z : Point$
  $\bullet\ betw(y, x, z) \land betw(z, y, x) \Rightarrow x = y \land y = z$
- $\forall\, x, y, z, w : Point$
  $\bullet\ betw(y, x, z) \land betw(z, x, w) \Rightarrow betw(y, x, w)$
- $\forall\, x, y : Region; z : Point \bullet x = y \Leftrightarrow (is\_in(z, x) \Leftrightarrow is\_in(z, y))$
- $\forall\, x, y : Region; z : Point$
  $\bullet\ subset(x, y) \Leftrightarrow is\_in(z, x) \Rightarrow is\_in(z, y)$
- $\forall\, x, y : Time \bullet precedes(x, y) \land precedes(y, x) \Rightarrow y = x$
- $\forall\, x, y, z : Time$
  $\bullet\ precedes(x, y) \land precedes(y, z) \Rightarrow precedes(x, z)$
- $\forall\, x, y : Time \bullet precedes(x, y) \lor precedes(y, x)$
- $\forall\, x, y : Time$
  $\bullet\ precedes\_neq(x, y) \Leftrightarrow precedes(x, y) \land \neg\, x = y$
- $\forall\, x, y : Fluent; t : Time$
  $\bullet\ x = y \Leftrightarrow value\_at(x, t) = value\_at(y, t)$
- $\forall\, x : Obj \bullet continuous(place(x))$
- $\forall\, x : Obj \bullet x = x$
- $\forall\, x : Obj \bullet place(x) = place(x)$
- $\forall\, x : Obj; y : Point; t : Time$
  $\bullet\ at(x, y, t) \Leftrightarrow value\_at(place(x), t) = y$

$\bullet \; \forall \, x : \mathit{Obj}; \, y, z : \mathit{Region}; \, t : \mathit{Time}$

$\quad \bullet \; \mathit{in}(x, y, t) \Leftrightarrow \mathit{is\_in}(\mathit{value\_at}(\mathit{place}(x), t), y)$

$\bullet \; \forall \, x : \mathit{Obj}; \, y, z : \mathit{Point}; \, \mathit{t\_1}, \mathit{t\_2} : \mathit{Time}$

$\quad \bullet \; \mathit{go\_1}(x, y, z, \mathit{t\_1}, \mathit{t\_2})$

$\quad \Leftrightarrow \mathit{at}(x, y, \mathit{t\_1}) \wedge \mathit{at}(x, z, \mathit{t\_2}) \wedge \mathit{precedes}(\mathit{t\_1}, \mathit{t\_2})$

$\bullet \; \forall \, x : \mathit{Obj}; \, y, z : \mathit{Region}; \, \mathit{t\_1}, \mathit{t\_2} : \mathit{Time}$

$\quad \bullet \; \mathit{go\_2}(x, y, z, \mathit{t\_1}, \mathit{t\_2})$

$\quad \Leftrightarrow \mathit{in}(x, y, \mathit{t\_1}) \wedge \mathit{in}(x, z, \mathit{t\_2}) \wedge \mathit{precedes}(\mathit{t\_1}, \mathit{t\_2})$

$\bullet \; \forall \, x : \mathit{Obj}; \, y : \mathit{Point}; \, \mathit{t\_1}, \mathit{t\_2}, \mathit{t\_3} : \mathit{Time}$

$\quad \bullet \; \mathit{stay\_1}(x, y, \mathit{t\_1}, \mathit{t\_3})$

$\quad \Leftrightarrow \mathit{precedes}(\mathit{t\_1}, \mathit{t\_2}) \wedge \mathit{precedes}(\mathit{t\_2}, \mathit{t\_3}) \Rightarrow \mathit{at}(x, y, \mathit{t\_2})$

$\bullet \; \forall \, x : \mathit{Obj}; \, y, z : \mathit{Region}; \, \mathit{t\_1}, \mathit{t\_2}, \mathit{t\_3} : \mathit{Time}$

$\quad \bullet \; \mathit{stay\_2}(x, y, \mathit{t\_1}, \mathit{t\_3})$

$\quad \Leftrightarrow \mathit{precedes}(\mathit{t\_1}, \mathit{t\_2}) \wedge \mathit{precedes}(\mathit{t\_2}, \mathit{t\_3}) \Rightarrow \mathit{in}(x, y, \mathit{t\_2})$

**end**

## A.2 Change in Temperature

**spec** CHANGE_IN_TEMPERATURE =

**sort** *Obj*

**sort** *Temp*

**sort** *TempRegion*

**sort** *Time*

**sort** *Fluent*

**pred** $\mathit{colder} : \mathit{Temp} \times \mathit{Temp}$

**pred** $\mathit{betw} : \mathit{Temp} \times \mathit{Temp} \times \mathit{Temp}$

**pred** $\mathit{is\_in} : \mathit{Temp} \times \mathit{TempRegion}$

**pred** $\mathit{subset} : \mathit{TempRegion} \times \mathit{TempRegion}$

**pred** $\mathit{precedes} : \mathit{Time} \times \mathit{Time}$

**pred** $\mathit{precedes\_neq} : \mathit{Time} \times \mathit{Time}$

**pred** $\mathit{continuous} : \mathit{Fluent}$

**op** $\mathit{temp} : \mathit{Obj} \rightarrow \mathit{Fluent}$

**op** $\mathit{value\_at} : \mathit{Fluent} \times \mathit{Time} \rightarrow \mathit{Temp}$

$\bullet \; \forall \, x, y : \mathit{Temp} \bullet \mathit{colder}(x, y) \wedge \mathit{colder}(y, x) \Rightarrow y = x$

$\bullet \; \forall \, x, y, z : \mathit{Temp} \bullet \mathit{colder}(x, y) \wedge \mathit{colder}(y, z) \Rightarrow \mathit{colder}(x, z)$

$\bullet \; \forall \, x, y : \mathit{Temp} \bullet \mathit{colder}(x, y) \vee \mathit{colder}(y, x)$

$\bullet \; \forall \, x, y, z : \mathit{Temp}$

$\bullet\ betw(y, x, z)$
$\Leftrightarrow (colder(x, y) \wedge colder(y, z))$
$\vee (colder(z, y) \wedge colder(y, x))$

$\bullet\ \forall\, x, y, z : Temp \bullet betw(y, x, z) \Rightarrow betw(y, z, x)$

$\bullet\ \forall\, x, y, z : Temp$
$\bullet\ betw(y, x, z) \wedge betw(z, y, x) \Rightarrow x = y \wedge y = z$

$\bullet\ \forall\, x, y, z, w : Temp$
$\bullet\ betw(y, x, z) \wedge betw(z, x, w) \Rightarrow betw(y, x, w)$

$\bullet\ \forall\, x, y : TempRegion; z : Temp$
$\bullet\ x = y \Leftrightarrow (is\_in(z, x) \Leftrightarrow is\_in(z, y))$

$\bullet\ \forall\, x, y : TempRegion; z : Temp$
$\bullet\ subset(x, y) \Leftrightarrow is\_in(z, x) \Rightarrow is\_in(z, y)$

$\bullet\ \forall\, x, y : Time \bullet precedes(x, y) \wedge precedes(y, x) \Rightarrow y = x$

$\bullet\ \forall\, x, y, z : Time$
$\bullet\ precedes(x, y) \wedge precedes(y, z) \Rightarrow precedes(x, z)$

$\bullet\ \forall\, x, y : Time \bullet precedes(x, y) \vee precedes(y, x)$

$\bullet\ \forall\, x, y : Time$
$\bullet\ precedes\_neq(x, y) \Leftrightarrow precedes(x, y) \wedge \neg\, x = y$

$\bullet\ \forall\, x, y : Fluent; t : Time$
$\bullet\ x = y \Leftrightarrow value\_at(x, t) = value\_at(y, t)$

$\bullet\ \forall\, x : Obj \bullet continuous(temp(x))$

$\bullet\ \forall\, x : Obj \bullet x = x$

$\bullet\ \forall\, x : Obj \bullet temp(x) = temp(x)$

**end**

## A.3 Change in Emotion

**spec** CHANGE_IN_EMOTION =

**sort** *Person*
**sort** *Val*
**sort** *Arous*
**sort** *Emot*
**sort** *EmotRegion*
**sort** *Time*
**sort** *Fluent*
**pred** $better : Val \times Val$
**pred** $betw\_val : Val \times Val \times Val$

**pred** $calmer : Arous \times Arous$

**pred** $betw\_arous : Arous \times Arous \times Arous$

**pred** $betw : Emot \times Emot \times Emot$

**pred** $is\_in : Emot \times EmotRegion$

**pred** $subset : EmotRegion \times EmotRegion$

**pred** $precedes : Time \times Time$

**pred** $precedes\_neq : Time \times Time$

**op** $min\_val : Val$

**op** $max\_val : Val$

**op** $min\_arous : Arous$

**op** $max\_arous : Arous$

**op** $val : Emot \to Val$

**op** $arous : Emot \to Arous$

**op** $emot : Person \to Fluent$

**op** $value\_at : Fluent \times Time \to Emot$

$\bullet \; \forall\, x, y : Val \bullet better(x, y) \wedge better(y, x) \Rightarrow y = x$

$\bullet \; \forall\, x, y, z : Val \bullet better(x, y) \wedge better(y, z) \Rightarrow better(x, z)$

$\bullet \; \forall\, x, y : Val \bullet better(x, y) \vee better(y, x)$

$\bullet \; \forall\, x : Val \bullet better(max\_val, x)$

$\bullet \; \forall\, x : Val \bullet better(x, min\_val)$

$\bullet \; \forall\, x, y, z : Val$

$\quad \bullet \; betw\_val(y, x, z)$

$\qquad \Leftrightarrow (better(x, y) \wedge better(y, z)) \vee (better(z, y) \wedge better(y, x))$

$\bullet \; \forall\, x, y : Arous \bullet calmer(x, y) \wedge calmer(y, x) \Rightarrow y = x$

$\bullet \; \forall\, x, y, z : Arous \bullet calmer(x, y) \wedge calmer(y, z) \Rightarrow calmer(x, z)$

$\bullet \; \forall\, x, y : Arous \bullet calmer(x, y) \vee calmer(y, x)$

$\bullet \; \forall\, x : Arous \bullet calmer(min\_arous, x)$

$\bullet \; \forall\, x : Arous \bullet calmer(x, max\_arous)$

$\bullet \; \forall\, x, y, z : Arous$

$\quad \bullet \; betw\_arous(y, x, z)$

$\qquad \Leftrightarrow (calmer(x, y) \wedge calmer(y, z))$

$\qquad\quad \vee (calmer(z, y) \wedge calmer(y, x))$

$\bullet \; \forall\, x, y : Emot \bullet x = y \Leftrightarrow val(x) = val(y) \wedge arous(x) = arous(y)$

$\bullet \; \forall\, x, y, z : Emot$

$\quad \bullet \; betw(y, x, z)$

$\qquad \Leftrightarrow betw\_val(val(y), val(x), val(z))$

$\qquad\quad \wedge betw\_arous(arous(y), arous(x), arous(z))$

- $\forall\, x, y, z : Emot \bullet betw(y, x, z) \Rightarrow betw(y, z, x)$
- $\forall\, x, y, z : Emot$
  - $betw(y, x, z) \wedge betw(z, y, x) \Rightarrow x = y \wedge y = z$
- $\forall\, x, y, z, w : Emot$
  - $betw(y, x, z) \wedge betw(z, x, w) \Rightarrow betw(y, x, w)$
- $\forall\, x, y : EmotRegion; z : Emot$
  - $x = y \Leftrightarrow (is\_in(z, x) \Leftrightarrow is\_in(z, y))$
- $\forall\, x, y : EmotRegion; z : Emot$
  - $subset(x, y) \Leftrightarrow is\_in(z, x) \Rightarrow is\_in(z, y)$
- $\forall\, x, y : Time \bullet precedes(x, y) \wedge precedes(y, x) \Rightarrow y = x$
- $\forall\, x, y, z : Time$
  - $precedes(x, y) \wedge precedes(y, z) \Rightarrow precedes(x, z)$
- $\forall\, x, y : Time \bullet precedes(x, y) \vee precedes(y, x)$
- $\forall\, x, y : Time$
  - $precedes\_neq(x, y) \Leftrightarrow precedes(x, y) \wedge \neg\, x = y$
- $\forall\, x, y : Fluent; t : Time$
  - $x = y \Leftrightarrow value\_at(x, t) = value\_at(y, t)$
- $\forall\, x : Person \bullet x = x$
- $\forall\, x : Person \bullet emot(x) = emot(x)$

**end**

## A.4 Inheritance

**spec** INHERITANCE =

**sort** *Person*
**sort** *Set*
**sort** *Heritable*
**sort** *Time*
**sort** *Fluent*
**pred** $ancestor : Person \times Person$
**pred** $betw : Person \times Person \times Person$
**pred** $is\_in : Person \times Set$
**pred** $subset : Set \times Set$
**pred** $precedes : Time \times Time$
**pred** $precedes\_neq : Time \times Time$
**pred** $continuous : Fluent$
**op** $owner : Heritable \rightarrow Fluent$

**op** $value\_at : Fluent \times Time \to Person$

$\bullet\ \forall\, x : Person \bullet ancestor(x, x)$

$\bullet\ \forall\, x, y : Person \bullet ancestor(x, y) \wedge ancestor(y, x) \Rightarrow x = y$

$\bullet\ \forall\, x, y, z : Person$

$\quad \bullet\ ancestor(x, y) \wedge ancestor(y, z) \Rightarrow ancestor(x, z)$

$\bullet\ \forall\, x, y, z : Person$

$\quad \bullet\ betw(y, x, z)$

$\quad\quad \Leftrightarrow (ancestor(x, y) \wedge ancestor(y, z))$

$\quad\quad\quad \vee (ancestor(z, y) \wedge ancestor(y, x))$

$\bullet\ \forall\, x, y, z : Person \bullet betw(y, x, z) \Rightarrow betw(y, z, x)$

$\bullet\ \forall\, x, y, z : Person$

$\quad \bullet\ betw(y, x, z) \wedge betw(z, y, x) \Rightarrow x = y \wedge y = z$

$\bullet\ \forall\, x, y, z, w : Person$

$\quad \bullet\ betw(y, x, z) \wedge betw(z, x, w) \Rightarrow betw(y, x, w)$

$\bullet\ \forall\, x, y : Set; z : Person \bullet x = y \Leftrightarrow (is\_in(z, x) \Leftrightarrow is\_in(z, y))$

$\bullet\ \forall\, x, y : Set; z : Person$

$\quad \bullet\ subset(x, y) \Leftrightarrow is\_in(z, x) \Rightarrow is\_in(z, y)$

$\bullet\ \forall\, x, y : Time \bullet precedes(x, y) \wedge precedes(y, x) \Rightarrow y = x$

$\bullet\ \forall\, x, y, z : Time$

$\quad \bullet\ precedes(x, y) \wedge precedes(y, z) \Rightarrow precedes(x, z)$

$\bullet\ \forall\, x, y : Time \bullet precedes(x, y) \vee precedes(y, x)$

$\bullet\ \forall\, x, y : Time$

$\quad \bullet\ precedes\_neq(x, y) \Leftrightarrow precedes(x, y) \wedge \neg\, x = y$

$\bullet\ \forall\, x, y : Fluent; t : Time$

$\quad \bullet\ x = y \Leftrightarrow value\_at(x, t) = value\_at(y, t)$

$\bullet\ \forall\, x : Heritable \bullet continuous(owner(x))$

$\bullet\ \forall\, x : Fluent; t1, t2 : Time$

$\quad \bullet\ precedes\_neq(t1, t2)$

$\quad\quad \Rightarrow ancestor(value\_at(x, t1), value\_at(x, t2))$

$\bullet\ \forall\, x : Heritable \bullet x = x$

$\bullet\ \forall\, x : Heritable \bullet owner(x) = owner(x)$

**end**

# Appendix B

## B.1 Experiment 1

**spec** GENERALISATION0 =

**sort** *Obj*

**sort** *Fluent*

**sort** *Time*

**sort** *Region_TempRegion*

**sort** *Point_Temp*

**pred** $betw : Point\_Temp \times Point\_Temp \times Point\_Temp$

**pred** $continuous : Fluent$

**pred** $is\_in : Point\_Temp \times Region\_TempRegion$

**op** $G\_G1841795 : Obj \to Fluent$

**pred** $precedes : Time \times Time$

**pred** $precedes\_neq : Time \times Time$

**pred** $subset : Region\_TempRegion \times Region\_TempRegion$

**op** $value\_at : Fluent \times Time \to Point\_Temp$

$\forall\, G\_G1847388 : Point\_Temp; G\_G1847297 : Point\_Temp;$

$G\_G1847206 : Point\_Temp$

$\bullet\ betw(G\_G1847206, G\_G1847297, G\_G1847388)$

$\Rightarrow betw(G\_G1847206, G\_G1847388, G\_G1847297)$

$\forall\, G\_G1846924 : Point\_Temp; G\_G1846833 : Point\_Temp;$

$G\_G1846742 : Point\_Temp$

$\bullet\ betw(G\_G1846742, G\_G1846833, G\_G1846924)$

$\wedge\ betw(G\_G1846924, G\_G1846742, G\_G1846833)$

$\Rightarrow G\_G1846833 = G\_G1846742 \wedge G\_G1846742 = G\_G1846924$

$\forall\, G\_G1846544 : Point\_Temp; G\_G1846411 : Point\_Temp;$

$G\_G1846320 : Point\_Temp; G\_G1846229 : Point\_Temp$

$\bullet\ betw(G\_G1846229, G\_G1846320, G\_G1846411)$

$\wedge\ betw(G\_G1846411, G\_G1846320, G\_G1846544)$

$\Rightarrow betw(G\_G1846229, G\_G1846320, G\_G1846544)$

$\forall\ G\_G1845911 : Point\_Temp; G\_G1845788 : Region\_TempRegion;$
$G\_G1845697 : Region\_TempRegion$

$\bullet\ G\_G1845697 = G\_G1845788$

$\Leftrightarrow (is\_in(G\_G1845911, G\_G1845697)$

$\Leftrightarrow is\_in(G\_G1845911, G\_G1845788))$

$\forall\ G\_G1845515 : Point\_Temp; G\_G1845272 : Region\_TempRegion;$
$G\_G1845181 : Region\_TempRegion$

$\bullet\ subset(G\_G1845181, G\_G1845272)$

$\Leftrightarrow is\_in(G\_G1845515, G\_G1845181)$

$\Rightarrow is\_in(G\_G1845515, G\_G1845272)$

$\forall\ G\_G1844850 : Time; G\_G1844759 : Time$

$\bullet\ precedes(G\_G1844759, G\_G1844850)$

$\wedge\ precedes(G\_G1844850, G\_G1844759)$

$\Rightarrow G\_G1844850 = G\_G1844759$

$\forall\ G\_G1844574 : Time; G\_G1844454 : Time; G\_G1844363 : Time$

$\bullet\ precedes(G\_G1844363, G\_G1844454)$

$\wedge\ precedes(G\_G1844454, G\_G1844574)$

$\Rightarrow precedes(G\_G1844363, G\_G1844574)$

$\forall\ G\_G1844058 : Time; G\_G1843967 : Time$

$\bullet\ precedes(G\_G1843967, G\_G1844058)$

$\vee\ precedes(G\_G1844058, G\_G1843967)$

$\forall\ G\_G1843244 : Time; G\_G1843153 : Time$

$\bullet\ precedes\_neq(G\_G1843153, G\_G1843244)$

$\Leftrightarrow precedes(G\_G1843153, G\_G1843244)$

$\wedge\ \neg\ G\_G1843153 = G\_G1843244$

$\forall\ G\_G1842864 : Time; G\_G1842608 : Fluent; G\_G1842517 : Fluent$

$\bullet\ G\_G1842517 = G\_G1842608$

$\Leftrightarrow value\_at(G\_G1842517, G\_G1842864)$

$= value\_at(G\_G1842608, G\_G1842864)$

$\forall\ G\_G1842270 : Obj \bullet continuous(G\_G1841795(G\_G1842270))$

$\forall\ G\_G1842010 : Obj \bullet G\_G1842010 = G\_G1842010$

$\forall\ G\_G1853981 : Obj$

$\bullet\ G\_G1841795(G\_G1853981) = G\_G1841795(G\_G1853981)$

**end**

%Generalisation0 mapping Change_In_Place to Change_In_Temperature Cost = 266

**view** MAPPING0_1 :
GENERALISATION0 **to** CHANGE_IN_PLACE =
*Obj* ↦ *Obj*, *Point_Temp* ↦ *Point*,
*Region_TempRegion* ↦ *Region*, *Time* ↦ *Time*,
*Fluent* ↦ *Fluent*, *betw* ↦ *betw*, *continuous* ↦ *continuous*,
*is_in* ↦ *is_in*, *G_G1841795* ↦ *place*, *precedes* ↦ *precedes*,
*precedes_neq* ↦ *precedes_neq*, *subset* ↦ *subset*,
*value_at* ↦ *value_at*
**end**

**view** MAPPING0_2 :
GENERALISATION0 **to** CHANGE_IN_TEMPERATURE =
*Obj* ↦ *Obj*, *Point_Temp* ↦ *Temp*,
*Region_TempRegion* ↦ *TempRegion*, *Time* ↦ *Time*,
*Fluent* ↦ *Fluent*, *betw* ↦ *betw*, *continuous* ↦ *continuous*,
*is_in* ↦ *is_in*, *G_G1841795* ↦ *temp*, *precedes* ↦ *precedes*,
*precedes_neq* ↦ *precedes_neq*, *subset* ↦ *subset*,
*value_at* ↦ *value_at*
**end**

**spec** CHANGE_IN_PLACE_TEMPERATURE =
**combine** *mapping0_1*, *mapping0_2*
**end**

## B.2 Experiment 2

**spec** GENERALISATION1 =
**sort** *Obj_Person*
**sort** *Fluent*
**sort** *Time*
**sort** *Region_EmotRegion*
**sort** *Point_Emot*
**pred** *betw* : *Point_Emot* × *Point_Emot* × *Point_Emot*
**pred** *is_in* : *Point_Emot* × *Region_EmotRegion*
**op** *G_G5351027* : *Obj_Person* → *Fluent*

$\textbf{pred}\ \ precedes : Time \times Time$

$\textbf{pred}\ \ precedes\_neq : Time \times Time$

$\textbf{pred}\ \ subset : Region\_EmotRegion \times Region\_EmotRegion$

$\textbf{op}\ \ value\_at : Fluent \times Time \to Point\_Emot$

$\forall\ G\_G5342093 : Point\_Emot; G\_G5342002 : Point\_Emot;$
$G\_G5341911 : Point\_Emot$
$\bullet\ betw(G\_G5341911, G\_G5342002, G\_G5342093)$
$\Rightarrow betw(G\_G5341911, G\_G5342093, G\_G5342002)$

$\forall\ G\_G5341629 : Point\_Emot; G\_G5341538 : Point\_Emot;$
$G\_G5341447 : Point\_Emot$
$\bullet\ betw(G\_G5341447, G\_G5341538, G\_G5341629)$
$\wedge\ betw(G\_G5341629, G\_G5341447, G\_G5341538)$
$\Rightarrow G\_G5341538 = G\_G5341447 \wedge G\_G5341447 = G\_G5341629$

$\forall\ G\_G5341249 : Point\_Emot; G\_G5341116 : Point\_Emot;$
$G\_G5341025 : Point\_Emot; G\_G5340934 : Point\_Emot$
$\bullet\ betw(G\_G5340934, G\_G5341025, G\_G5341116)$
$\wedge\ betw(G\_G5341116, G\_G5341025, G\_G5341249)$
$\Rightarrow betw(G\_G5340934, G\_G5341025, G\_G5341249)$

$\forall\ G\_G5340616 : Point\_Emot; G\_G5340493 : Region\_EmotRegion;$
$G\_G5340402 : Region\_EmotRegion$
$\bullet\ G\_G5340402 = G\_G5340493$
$\Leftrightarrow (is\_in(G\_G5340616, G\_G5340402)$
$\Leftrightarrow is\_in(G\_G5340616, G\_G5340493))$

$\forall\ G\_G5340220 : Point\_Emot; G\_G5339977 : Region\_EmotRegion;$
$G\_G5339886 : Region\_EmotRegion$
$\bullet\ subset(G\_G5339886, G\_G5339977)$
$\Leftrightarrow is\_in(G\_G5340220, G\_G5339886)$
$\Rightarrow is\_in(G\_G5340220, G\_G5339977)$

$\forall\ G\_G5339359 : Time; G\_G5339239 : Time; G\_G5339148 : Time$
$\bullet\ precedes(G\_G5339148, G\_G5339239)$
$\wedge\ precedes(G\_G5339239, G\_G5339359)$
$\Rightarrow precedes(G\_G5339148, G\_G5339359)$

$\forall\ G\_G5338843 : Time; G\_G5338752 : Time$
$\bullet\ precedes(G\_G5338752, G\_G5338843)$
$\vee\ precedes(G\_G5338843, G\_G5338752)$

$\forall\ G\_G5338029 : Time; G\_G5337938 : Time$
$\bullet\ precedes\_neq(G\_G5337938, G\_G5338029)$

$\Leftrightarrow precedes(G\_G5337938, G\_G5338029)$

$\wedge \neg\ G\_G5337938 = G\_G5338029$

$\forall\ G\_G5337649 : Time; G\_G5337393 : Fluent;$

$G\_G5337302 : Fluent$

$\bullet\ G\_G5337302 = G\_G5337393$

$\Leftrightarrow value\_at(G\_G5337302, G\_G5337649)$

$= value\_at(G\_G5337393, G\_G5337649)$

$\forall\ G\_G5336934 : Obj\_Person \bullet G\_G5336934 = G\_G5336934$

$\forall\ G\_G5350811 : Obj\_Person$

$\bullet\ G\_G5351027(G\_G5350811) = G\_G5351027(G\_G5350811)$

**end**

%Generalisation1 mapping Change_In_Place to Change_In_Emotion Cost = 294

**view** MAPPING1_1 :

GENERALISATION1 **to** CHANGE_IN_PLACE =

$Obj\_Person \mapsto Obj, Point\_Emot \mapsto Point,$

$Region\_EmotRegion \mapsto Region, Time \mapsto Time,$

$Fluent \mapsto Fluent, betw \mapsto betw, is\_in \mapsto is\_in,$

$G\_G5351027 \mapsto place, precedes \mapsto precedes,$

$precedes\_neq \mapsto precedes\_neq, subset \mapsto subset,$

$value\_at \mapsto value\_at$

**end**

**view** MAPPING1_2 :

GENERALISATION1 **to** CHANGE_IN_EMOTION =

$Obj\_Person \mapsto Person, Point\_Emot \mapsto Emot,$

$Region\_EmotRegion \mapsto EmotRegion, Time \mapsto Time,$

$Fluent \mapsto Fluent, betw \mapsto betw, is\_in \mapsto is\_in,$

$G\_G5351027 \mapsto emot, precedes \mapsto precedes,$

$precedes\_neq \mapsto precedes\_neq, subset \mapsto subset,$

$value\_at \mapsto value\_at$

**end**

**spec** CHANGE_IN_PLACE_EMOTION =

**combine** *mapping1_1*, *mapping1_2*

**end**

## B.3 Experiment 3

**spec** $\text{GENERALISATION2} =$

**sort** *Obj_Heritable*

**sort** *Time*

**sort** *Region_Set*

**sort** *Point_Person*

**pred** $betw : Point\_Person \times Point\_Person \times Point\_Person$

**sort** *Fluent*

**pred** $continuous : Fluent$

**pred** $is\_in : Point\_Person \times Region\_Set$

**op** $G\_G5280638 : Obj\_Heritable \to Fluent$

**pred** $precedes : Time \times Time$

**pred** $precedes\_neq : Time \times Time$

**pred** $subset : Region\_Set \times Region\_Set$

$\forall\ G\_G5285974 : Point\_Person;\ G\_G5285883 : Point\_Person;$
$G\_G5285792 : Point\_Person$
$\bullet\ betw(G\_G5285792, G\_G5285883, G\_G5285974)$
$\quad \Rightarrow betw(G\_G5285792, G\_G5285974, G\_G5285883)$

$\forall\ G\_G5285510 : Point\_Person;\ G\_G5285419 : Point\_Person;$
$G\_G5285328 : Point\_Person$
$\bullet\ betw(G\_G5285328, G\_G5285419, G\_G5285510)$
$\quad \wedge betw(G\_G5285510, G\_G5285328, G\_G5285419)$
$\quad \Rightarrow G\_G5285419 = G\_G5285328 \wedge G\_G5285328 = G\_G5285510$

$\forall\ G\_G5285130 : Point\_Person;\ G\_G5284997 : Point\_Person;$
$G\_G5284906 : Point\_Person;\ G\_G5284815 : Point\_Person$
$\bullet\ betw(G\_G5284815, G\_G5284906, G\_G5284997)$
$\quad \wedge betw(G\_G5284997, G\_G5284906, G\_G5285130)$
$\quad \Rightarrow betw(G\_G5284815, G\_G5284906, G\_G5285130)$

$\forall\ G\_G5284497 : Point\_Person;\ G\_G5284374 : Region\_Set;$
$G\_G5284283 : Region\_Set$
$\bullet\ G\_G5284283 = G\_G5284374$
$\quad \Leftrightarrow (is\_in(G\_G5284497, G\_G5284283)$
$\qquad \Leftrightarrow is\_in(G\_G5284497, G\_G5284374))$

$\forall\ G\_G5284101 : Point\_Person;\ G\_G5283858 : Region\_Set;$
$G\_G5283767 : Region\_Set$
$\bullet\ subset(G\_G5283767, G\_G5283858)$
$\quad \Leftrightarrow is\_in(G\_G5284101, G\_G5283767)$

$\Rightarrow is\_in(G\_G5284101, G\_G5283858)$

$\forall\ G\_G5283436 : Time; G\_G5283345 : Time$

$\bullet\ precedes(G\_G5283345, G\_G5283436)$

$\wedge\ precedes(G\_G5283436, G\_G5283345)$

$\Rightarrow G\_G5283436 = G\_G5283345$

$\forall\ G\_G5283160 : Time; G\_G5283040 : Time; G\_G5282949 : Time$

$\bullet\ precedes(G\_G5282949, G\_G5283040)$

$\wedge\ precedes(G\_G5283040, G\_G5283160)$

$\Rightarrow precedes(G\_G5282949, G\_G5283160)$

$\forall\ G\_G5282644 : Time; G\_G5282553 : Time$

$\bullet\ precedes(G\_G5282553, G\_G5282644)$

$\vee\ precedes(G\_G5282644, G\_G5282553)$

$\forall\ G\_G5281830 : Time; G\_G5281739 : Time$

$\bullet\ precedes\_neq(G\_G5281739, G\_G5281830)$

$\Leftrightarrow precedes(G\_G5281739, G\_G5281830)$

$\wedge\ \neg\ G\_G5281739 = G\_G5281830$

$\forall\ G\_G5281113 : Obj\_Heritable$

$\bullet\ continuous(G\_G5280638(G\_G5281113))$

$\forall\ G\_G5280853 : Obj\_Heritable \bullet G\_G5280853 = G\_G5280853$

$\forall\ G\_G5292747 : Obj\_Heritable$

$\bullet\ G\_G5280638(G\_G5292747) = G\_G5280638(G\_G5292747)$

**end**

%Generalisation2 mapping Change_In_Place to Inheritance Cost = 288

**view** MAPPING2_1 :

GENERALISATION2 **to** CHANGE_IN_PLACE =

$Obj\_Heritable \mapsto Obj, Point\_Person \mapsto Point,$

$Region\_Set \mapsto Region, Time \mapsto Time, Fluent \mapsto Fluent,$

$betw \mapsto betw, continuous \mapsto continuous, is\_in \mapsto is\_in,$

$G\_G5280638 \mapsto place, precedes \mapsto precedes,$

$precedes\_neq \mapsto precedes\_neq, subset \mapsto subset$

**end**

**view** MAPPING2_2 :

GENERALISATION2 **to** INHERITANCE =

$Obj\_Heritable \mapsto Heritable, Point\_Person \mapsto Person,$

$Region\_Set \mapsto Set$, $Time \mapsto Time$, $Fluent \mapsto Fluent$,
$betw \mapsto betw$, $continuous \mapsto continuous$, $is\_in \mapsto is\_in$,
$G\_G5280638 \mapsto owner$, $precedes \mapsto precedes$,
$precedes\_neq \mapsto precedes\_neq$, $subset \mapsto subset$

**end**

**spec** CHANGE_IN_PLACE_INHERITANCE =
**combine** *mapping2_1*, *mapping2_2*

**end**

## B.4 Experiment 4

**spec** GENERALISATION3 =

**sort** $Obj\_Person$
**sort** $Fluent$
**sort** $Time$
**sort** $TempRegion\_EmotRegion$
**sort** $Temp\_Emot$
**sort** $Temp\_Arous$
**pred** $betw : Temp\_Emot \times Temp\_Emot \times Temp\_Emot$
**pred** $G\_G9476510 : Temp\_Arous \times Temp\_Arous$
**pred** $is\_in : Temp\_Emot \times TempRegion\_EmotRegion$
**pred** $precedes : Time \times Time$
**pred** $precedes\_neq : Time \times Time$
**pred** $subset : TempRegion\_EmotRegion \times TempRegion\_EmotRegion$
**op** $G\_G9483970 : Obj\_Person \to Fluent$
**op** $value\_at : Fluent \times Time \to Temp\_Emot$

$\forall\, G\_G9477582 : Temp\_Arous; G\_G9477491 : Temp\_Arous$
$\bullet\; G\_G9476510(G\_G9477491, G\_G9477582)$
$\wedge\; G\_G9476510(G\_G9477582, G\_G9477491)$
$\Rightarrow G\_G9477582 = G\_G9477491$

$\forall\, G\_G9476939 : Temp\_Arous; G\_G9476848 : Temp\_Arous$
$\bullet\; G\_G9476510(G\_G9476848, G\_G9476939)$
$\vee\; G\_G9476510(G\_G9476939, G\_G9476848)$

$\forall\, G\_G9476180 : Temp\_Emot; G\_G9476089 : Temp\_Emot;$
$G\_G9475998 : Temp\_Emot$
$\bullet\; betw(G\_G9475998, G\_G9476089, G\_G9476180)$

$\Rightarrow betw(G\_G9475998, G\_G9476180, G\_G9476089)$

$\forall\ G\_G9475716 : Temp\_Emot; G\_G9475625 : Temp\_Emot;$

$G\_G9475534 : Temp\_Emot$

$\bullet\ betw(G\_G9475534, G\_G9475625, G\_G9475716)$

$\wedge\ betw(G\_G9475716, G\_G9475534, G\_G9475625)$

$\Rightarrow G\_G9475625 = G\_G9475534 \wedge G\_G9475534 = G\_G9475716$

$\forall\ G\_G9475336 : Temp\_Emot; G\_G9475203 : Temp\_Emot;$

$G\_G9475112 : Temp\_Emot; G\_G9475021 : Temp\_Emot$

$\bullet\ betw(G\_G9475021, G\_G9475112, G\_G9475203)$

$\wedge\ betw(G\_G9475203, G\_G9475112, G\_G9475336)$

$\Rightarrow betw(G\_G9475021, G\_G9475112, G\_G9475336)$

$\forall\ G\_G9474703 : Temp\_Emot; G\_G9474580 : TempRegion\_EmotRegion;$

$G\_G9474489 : TempRegion\_EmotRegion$

$\bullet\ G\_G9474489 = G\_G9474580$

$\Leftrightarrow (is\_in(G\_G9474703, G\_G9474489)$

$\Leftrightarrow is\_in(G\_G9474703, G\_G9474580))$

$\forall\ G\_G9474307 : Temp\_Emot; G\_G9474064 : TempRegion\_EmotRegion;$

$G\_G9473973 : TempRegion\_EmotRegion$

$\bullet\ subset(G\_G9473973, G\_G9474064)$

$\Leftrightarrow is\_in(G\_G9474307, G\_G9473973)$

$\Rightarrow is\_in(G\_G9474307, G\_G9474064)$

$\forall\ G\_G9473642 : Time; G\_G9473551 : Time$

$\bullet\ precedes(G\_G9473551, G\_G9473642)$

$\wedge\ precedes(G\_G9473642, G\_G9473551)$

$\Rightarrow G\_G9473642 = G\_G9473551$

$\forall\ G\_G9473366 : Time; G\_G9473246 : Time; G\_G9473155 : Time$

$\bullet\ precedes(G\_G9473155, G\_G9473246)$

$\wedge\ precedes(G\_G9473246, G\_G9473366)$

$\Rightarrow precedes(G\_G9473155, G\_G9473366)$

$\forall\ G\_G9472850 : Time; G\_G9472759 : Time$

$\bullet\ precedes(G\_G9472759, G\_G9472850)$

$\vee\ precedes(G\_G9472850, G\_G9472759)$

$\forall\ G\_G9472036 : Time; G\_G9471945 : Time$

$\bullet\ precedes\_neq(G\_G9471945, G\_G9472036)$

$\Leftrightarrow precedes(G\_G9471945, G\_G9472036)$

$\wedge \neg\ G\_G9471945 = G\_G9472036$

$\forall\ G\_G9471656 : Time; G\_G9471400 : Fluent;$

$G\_G9471309 : Fluent$
$\bullet\ G\_G9471309 = G\_G9471400$
$\Leftrightarrow value\_at(G\_G9471309, G\_G9471656)$
$= value\_at(G\_G9471400, G\_G9471656)$
$\forall\ G\_G9470941 : Obj\_Person \bullet G\_G9470941 = G\_G9470941$
$\forall\ G\_G9483754 : Obj\_Person$
$\bullet\ G\_G9483970(G\_G9483754) = G\_G9483970(G\_G9483754)$
**end**

%Generalisation3 mapping Change_In_Temperature to Change_In_Emotion Cost = 92

**view** MAPPING3_1 :
GENERALISATION3 **to** CHANGE_IN_TEMPERATURE =
$Obj\_Person \mapsto Obj$, $Fluent \mapsto Fluent$, $Time \mapsto Time$,
$TempRegion\_EmotRegion \mapsto TempRegion$,
$Temp\_Emot \mapsto Temp$, $Temp\_Arous \mapsto Temp$, $betw \mapsto betw$,
$G\_G9476510 \mapsto colder$, $is\_in \mapsto is\_in$, $precedes \mapsto precedes$,
$precedes\_neq \mapsto precedes\_neq$, $subset \mapsto subset$,
$G\_G9483970 \mapsto temp$, $value\_at \mapsto value\_at$
**end**

**view** MAPPING3_2 :
GENERALISATION3 **to** CHANGE_IN_EMOTION =
$Obj\_Person \mapsto Person$, $Fluent \mapsto Fluent$, $Time \mapsto Time$,
$TempRegion\_EmotRegion \mapsto EmotRegion$,
$Temp\_Emot \mapsto Emot$, $Temp\_Arous \mapsto Arous$, $betw \mapsto betw$,
$G\_G9476510 \mapsto calmer$, $is\_in \mapsto is\_in$, $precedes \mapsto precedes$,
$precedes\_neq \mapsto precedes\_neq$, $subset \mapsto subset$,
$G\_G9483970 \mapsto emot$, $value\_at \mapsto value\_at$
**end**

**spec** CHANGE_IN_TEMPERATURE_EMOTION =
**combine** *mapping3_1*, *mapping3_2*
**end**

# Bibliography


F Bou, M Eppe, E Plaza, and M Schorlemmer. D2. 1: Reasoning with amalgams. Technical report, Technical report, COINVENT Project. http://www. coinventproject. eu/fileadmin/publications, 2014.

Margaret M Bradley, Mark K Greenwald, Margaret C Petry, and Peter J Lang. Remembering pictures: pleasure and arousal in memory. *Journal of experimental psychology: Learning, Memory, and Cognition*, 18(2):379, 1992.

Richard S Cook, Paul Kay, and Terry Regier. The world color survey database. *Handbook of categorization in cognitive science*, pages 223–241, 2005.

William Croft and D Alan Cruse. *Cognitive linguistics*. Cambridge University Press, 2004.

Ernest Davis. *Representations of commonsense knowledge*. Morgan Kaufmann, 2014.

Gilles Fauconnier. *Mental spaces: Aspects of meaning construction in natural language*. Cambridge University Press, 1994.

Gilles Fauconnier and Mark Turner. *The way we think: Conceptual blending and the mind's hidden complexities*. Basic Books, 2008.

Chiara Fedriani. Experiential metaphors in latin: feelings were containers, movements and things possessed1. *Transactions of the Philological Society*, 109(3):307–326, 2011.

Peter Gärdenfors. *Conceptual spaces: The geometry of thought*. MIT press, 2004.

Peter Gärdenfors. *The geometry of meaning: Semantics based on conceptual spaces*. MIT Press, 2014.

Joseph Goguen. Mathematical models of cognitive space and time. In *Reasoning and Cognition: Proc. of the Interdisciplinary Conference on Reasoning and Cognition*, pages 125–128, 2006.

Jeffrey Steven Gruber. *Studies in lexical relations.* PhD thesis, Massachusetts Institute of Technology, 1965.

Markus Guhe, Alison Pease, Alan Smaill, Maricarmen Martinez, Martin Schmidt, Helmar Gust, Kai-Uwe Kühnberger, and Ulf Krumnack. A computational account of conceptual blending in basic mathematics. *Cognitive Systems Research*, 12(3):249–265, 2011.

Helmar Gust, Kai-Uwe Kühnberger, and Ute Schmid. Metaphors and heuristic-driven theory projection (hdtp). *Theoretical Computer Science*, 354(1):98–117, 2006.

Edward V Huntington. Inter-relations among the four principal types of order. *Transactions of the American Mathematical Society*, 38(1):1–9, 1935.

Carroll E Izard, Deborah Z Libero, Priscilla Putnam, and O Maurice Haynes. Stability of emotion experiences and their relations to traits of personality. *Journal of personality and social psychology*, 64(5):847, 1993.

Ray Jackendoff. *Semantics and cognition*, volume 8. MIT press, 1983.

Ray Jackendoff. *Semantic structures*, volume 18. MIT press, 1992.

Ray Jackendoff and David Aaron. More than cool reason: A field guide to poetic metaphor by george lakoff and mark turner (review). *Language*, 67(2):320–338, 1991.

Gerhard Jäger. Natural color categories are convex sets. In *Logic, language and meaning*, pages 11–20. Springer, 2010.

Mark Johnson. *The body in the mind: The bodily basis of meaning, imagination, and reason*. University of Chicago Press, 2013.

George Lakoff. *Women, fire, and dangerous things: What categories reveal about the mind*. Cambridge Univ Press, 1990.

George Lakoff and Mark Johnson. *Metaphors we live by*. University of Chicago press, 2008.

George Lakoff and Rafael E Núñez. *Where mathematics comes from: How the embodied mind brings mathematics into being*. Basic books, 2000.

Ronald W Langacker. *Foundations of cognitive grammar: Theoretical prerequisites*, volume 1. Stanford university press, 1987.

Hector Levesque, Fiora Pirri, and Ray Reiter. Foundations for the situation calculus, 1998.

Albert Mehrabian. Basic dimensions for a general psychological theory implications for personality, social, environmental, and developmental studies. 1980.

M Mohrmann, Eppe M Confalonieri, Maclean E, and Winterstein. Coinvent blend ui. `http://server.coinvent-project.eu/static/blendui/index.html`. Accessed: 2016-07-31.

Till Mossakowski, Christian Maeder, and Klaus Lüttich. The heterogeneous tool set, hets. In *International Conference on Tools and Algorithms for the Construction and Analysis of Systems*, pages 519–522. Springer, 2007.

Peter D Mosses. *CASL reference manual: The complete documentation of the common algebraic specification language*, volume 2960. Springer, 2004.

Thomas Edward Payne. *Describing morphosyntax: A guide for field linguists*. Cambridge University Press, 1997.

Robert Plutchik. The nature of emotions human emotions have deep evolutionary roots, a fact that may explain their complexity and provide tools for clinical practice. *American Scientist*, 89(4):344–350, 2001.

Jonathan Posner, James A Russell, and Bradley S Peterson. The circumplex model of affect: An integrative approach to affective neuroscience, cognitive development, and psychopathology. *Development and psychopathology*, 17(03):715–734, 2005.

David C Rubin and Jennifer M Talarico. A comparison of dimensional models of emotion: Evidence from emotions, prototypical events, autobiographical memories, and words. *Memory*, 17(8):802–808, 2009.

Marco Schorlemmer, Alan Smaill, Kai-Uwe Kühnberger, Oliver Kutz, Simon Colton, Emilios Cambouropoulos, and Alison Pease. Coinvent: Towards a computational concept invention theory. 2014.

Angela Schwering, Ulf Krumnack, Kai-Uwe Kühnberger, and Helmar Gust. Syntactic principles of heuristic-driven theory projection. *Cognitive Systems Research*, 10(3):251–269, 2009.

Leon Stassen. *Predicative possession*. Oxford University Press, 2009.

Leonard Talmy. *Toward a cognitive semantics*, volume 1. MIT press, 2000.

Michael Thielscher. Introduction to the fluent calculus. 1998.

Ning Yu. *The contemporary theory of metaphor: A perspective from Chinese*, volume 1. John Benjamins Publishing, 1998.